# Course C9, Metals and Alloys
# Answers to Examples Class

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1. A high silicon concentration prevents the precipitation of cementite during the bainite transformation. The microstructure at the transformation temperature then consists of just bainitic ferrite and residual austenite. The transformation is diffusionless, but any excess carbon in the ferrite soon becomes partitioned into the austenite, which is thereby enriched.

   Such a transformation must stop when the carbon concentration of the residual austenite, *i.e.* $x_\gamma$ equals that given by the $T_0$ curve. From mass balance,

   $$V_\alpha x_\alpha + V_\gamma x_\gamma = \overline{x}$$

   where $V_\alpha$ is the volume fraction of ferrite, $V_\gamma$ is the volume fraction of residual austenite at the transformation temperature with $V_\gamma = 1 - V_\alpha$ and $\overline{x}$ is the average carbon concentration in the alloy. Take $x_\alpha$ to be zero since the solubility of carbon in ferrite is negligible, and set $x_\gamma = x_{T_0}$ in which case,

   $$V_\alpha = 1 - \frac{\overline{x}}{x_{T_0}} \equiv 1 - \frac{0.40}{0.70} = 0.43 \qquad \text{and} \qquad V_\gamma = 0.57$$

   Some of the residual austenite will transform to martensite on cooling to ambient temperature (25 °C). The martensite–start temperature $M_S$ of the carbon–enriched austenite is given by:

   $$M_s = 561 - 474 \times 0.70 - 33 \times 0.70 - 17 \times 0.8 - 17 \times 1.8 - 21 \times 0.25 = 157\,^\circ\mathrm{C}$$

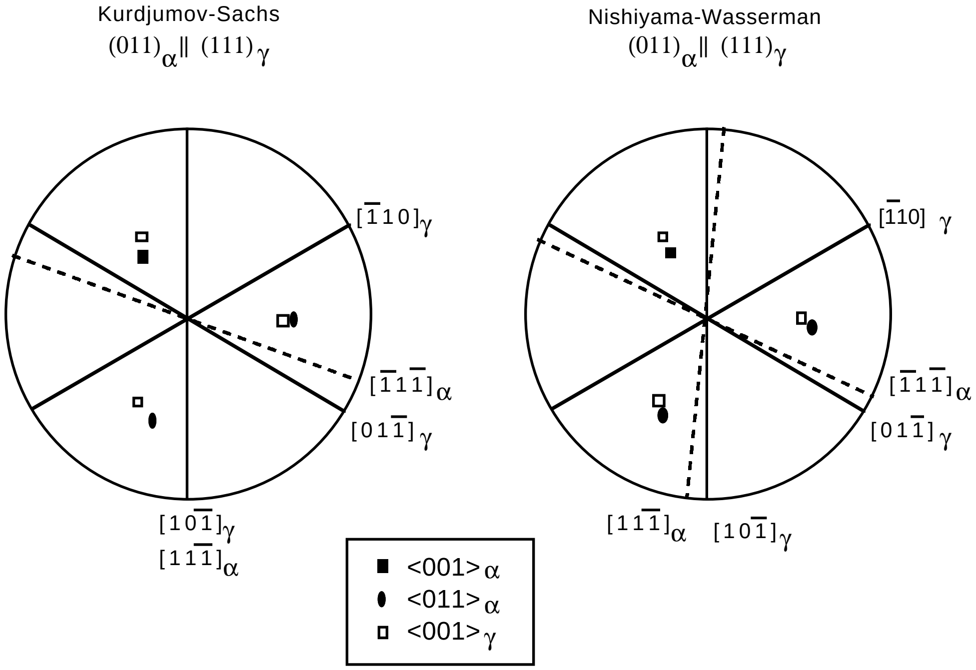
   From the Koistinen and Marburger equation, the volume fraction of martensite formed by cooling to 25 °C is therefore

   $$0.57 \times [1 - \exp\{-0.011(157 - 25)\}] = 0.44$$

   To summarise,

   $$V_\alpha = 0.43 \qquad V_\gamma = 0.13 \qquad V_{\alpha'} = 0.44$$

2. The sketch below shows that an appropriate rotation of 5.26° about $[011]_\alpha \equiv [111]_\gamma$ generates one orientation from the other. These orientation relationships are closely related because they both originate from the Bain Strain.



3. Suppose that the austenite is represented as a sphere with its unit cell edges denoted by the vectors $\mathbf{a_i}$ with $i = 1, 2, 3$, as illustrated in Figure 1a,b. The Bain strain changes the sphere into an ellipsoid of revolution about $\mathbf{a_1}$. There are no lines in the $(0\ 0\ 1)_\gamma$ plane which are undistorted. However, it is possible to find lines such as $wx$ and $yz$ are undistorted by the deformation, but are rotated to the new positions $w'x'$ and $y'z'$. One of these undistorted lines can be converted into an invariant–line by adding a rigid body rotation as illustrated in Figure 1c.

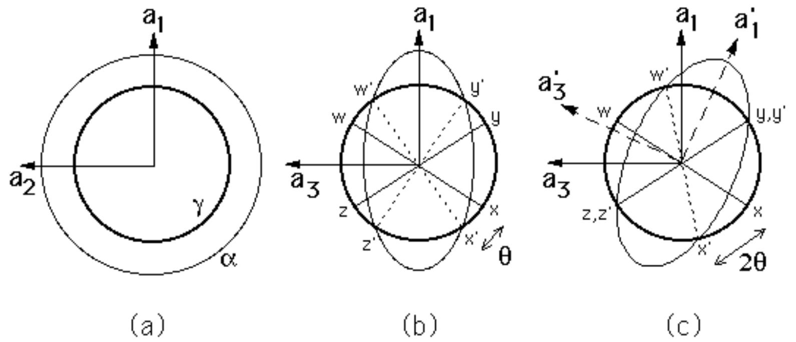


Figure 1: (a) and (b) show the effect of the Bain strain on austenite, which when undeformed is represented as a sphere of diameter $wx = yz$ in three–dimensions. The strain transforms it to an ellipsoid of revolution. (c) shows the invariant–line strain obtained by combining the Bain strain with a rigid body rotation.

However, there is no possible rotation which can create two non–parallel invariant–lines, *i.e.* an invariant–plane. Therefore, austenite and martensite in steel cannot ever be joined at an interface which is fully coherent and stress–free.

4. Discuss the aluminium–lithium system where ordered $Al_3Li$ precipitates in the cubic close–packed matrix (a disordered solid solution). Similarly, the $Ni_3Al$ or $Ni_3Ti$ $\gamma'$ precipitates in the cubic close–packed $\gamma$ disordered–matrix in the case of the nickel base superalloys. In both cases, the precipitates have a primitive cubic lattice with a cube–cube orientation relationship with the matrix.

   The Burgers vector for slip dislocations in a cubic close–packed crystal is $\frac{a}{2} < 1\ 1\ 0 >$. This is not, however, a lattice vector of the ordered precipitate; the corresponding lattice vector in the primitive cubic lattice is in fact $a < 1\ 1\ 0 >$. If the matrix dislocation enters the precipitate, it will leave in its trail a fault in the atomic order inside the precipitate. The fault can be corrected by the passage of a second dislocation with an identical Burgers vector. It is necessary therefore for a pair of appropriate lattice dislocations to enter the precipitate in order to avoid the faulting. Such pairs are known as superdislocations; the need for these superdislocations hardens the material, a phenomenon known as *order hardening.*

   The energy of a dislocation depends on the elastic modulus. The ordered precipitate is likely to have a different modulus when compared with the matrix. Additional work therefore has to be done to move dislocations across the matrix–precipitate interface, causing hardening.

   The state of coherency can be controlled by adjusting the chemical composition such that there the lattice parameters of the precipitate and matrix phases are identical at the service temperature. This can be done using ternary alloying additions, for example by adding both titanium and aluminium to a nickel–base superalloy. Thus, the parameter of the $Ni_3(Al,Ti)$ $\gamma'$ can be altered by adjusting the ratio of aluminium to titanium.