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OPTIMAL DESIGN AND ANALYSIS OF AUTOMOTIVE COMPOSITE DRIVE SHAFT

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ABSTRACT

The overall objective of this paper is to design and analyze a composite drive shaft for power transmission applications. A one-piece drive shaft for rear wheel drive automobile was designed optimally using E-Glass/Epoxy and High modulus (HM) Carbon/Epoxy composites. In this paper a Genetic Algorithm (GA) has been successfully applied to minimize the weight of shaft which is subjected to the constraints such as torque transmission, torsional buckling capacities and fundamental natural frequency. The results of GA are used to perform static and buckling analysis using ANSYS software. The results show the stacking sequence of shaft strongly affects buckling torque.

Keywords: drive shaft; GA; constraints; ANSYS software; stacking sequence

1. INTRODUCTION

Many methods are available at present for the design optimization of structural systems and these methods based on mathematical programming techniques involving gradient search and direct search. These methods assume that the design variables are continuous. But in practical structural engineering optimization, almost all the design variables are discrete. This is due to the availability of components in standard sizes and constraints due to construction and manufacturing practices.Beard more et.al[1] explained the potential for composites in structural automotive applications from a structural point of view. Andrew Pollard[2] proposed the polymer Matrix composites in driveline applications. The working of genetic algorithm is explained by Goldberg[3] based on natural genetics has been used in this work. In the previous study by the authors[4], a GA was applied for the design optimization of steel and composite leaf springs.

In the present work an attempt is made to evaluate the suitability of composite material such as E-Glass/Epoxy and HM-Carbon/Epoxy for the purpose of automotive transmission applications. A one-piece composite drive shaft for rear wheel drive automobile is optimally designed and analyzed using GA and ANSYS software respectively for E-Glass/Epoxy and HM-Carbon/Epoxy composites with the objective of minimization of weight of the shaft which is subjected to the constraints such as torque transmission, torsional buckling strength capabilities and natural bending frequency.

2. SPECIFICATION OF THE PROBLEM

The torque transmission capability of the drive shaft for passenger cars, small trucks, and vans should be larger than 3,500 Nm and fundamental natural bending frequency of the shaft should be higher than 6,500 rpm to avoid whirling vibration. The outer diameter (do) should not exceed 100 mm due to space limitations and here do is taken as 90 mm. The drive shaft of transmission system was designed optimally to the specified design requirements[5].

3. DESIGN OF COMPOSITE DRIVE SHAFT

3.1 Assumptions

The shaft rotates at a constant speed about its longitudinal axis. The shaft has a uniform, circular cross section. The shaft is perfectly balanced, i.e., at every cross section, the mass center coincides with the geometric center. All damping and nonlinear effects are excluded. The stress-strain relationship for composite material is linear & elastic; hence, Hook's law is applicable for composite materials. Since lamina is thin and no out-of-plane loads are applied, it is considered as under the plane stress

3.2 Selection of Cross-Section and Materials

The E-Glass/Epoxy and HM Carbon/Epoxy materials are selected for composite drive shaft. Since, composites are highly orthotropic and their fractures were not fully studied. The factor of safety was taken as 2 and the fiber volume fraction as 0.6.

3.3. Torque transmission capacity of the composite drive shaft
3.3.1. Stress-Strain Relationship for Unidirectional Lamina

The lamina is thin and if no out-of-plane loads are applied, it is considered as the plane stress problem. Hence, it is possible to reduce the 3-D problem into 2-D problem. For unidirectional 2-D lamina, the stress-strain relation ship in terms of physical material direction is given by:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix} \quad (1)$$

The matrix Q is referred as the reduced stiffness matrix for the layer and its terms are given by

$$Q_{11}=\frac{E_{11}}{1-\nu_{12}\nu_{21}} \;;\quad Q_{12}=\frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}} \;;$$

$$Q_{22}=\frac{E_{22}}{1-\nu_{12}\nu_{21}} \;;\quad Q_{66}=G_{12}$$

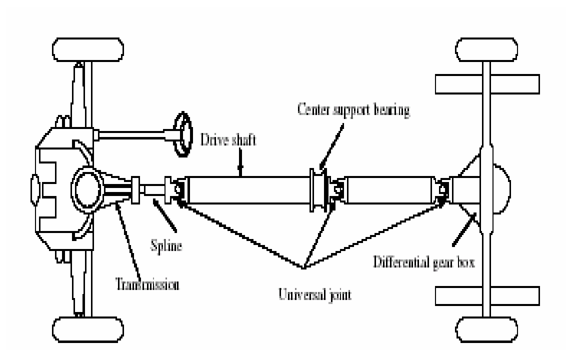

Fig.1. Conventional two-piece drive shaft arrangement for rear wheel vehicle driving system [6]

Table 1. Mechanical properties for each lamina of the laminate

	E-Glass/Epoxy	HM carbon/ Epoxy
E_{11} (GPa)	50.0	190.0
E_{22} (GPa)	12.0	7.7
G_{12} (GPa)	5.6	4.2
ν_{12}	0.3	0.3
$\sigma^T_1=\sigma^C_1$ (MPa)	800.0	870.0
$\sigma^T_2=\sigma^C_2$ (MPa)	40.0	54.0
τ_{12} (MPa)	72.0	30.0
ρ (Kg/m^3)	2000.0	1600.0

For an angle-ply lamina, where fibers are oriented at an angle with the positive X-axis (Longitudinal axis of shaft), the stress strain relationship is given by[7],

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}\overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}}\\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}}\\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} \quad (2)$$

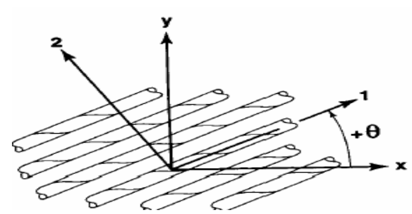

Fig. 2. principal materials axes from x-y axes

For a symmetric laminate, the B matrix vanishes and the in plane and bending stiff-nesses are uncoupled.

$$A_{ij}=\sum_{k=1}^{n}(\overline{Q}_{ij})_k(h_k-h_{k-1}) \;;\quad B_{ij}=\frac{1}{2}\sum_{k=1}^{n}(\overline{Q}_{ij})_k(h_k^2-h_{k-1}^2) \;;\quad D_{ij}=\frac{1}{3}\sum_{k=1}^{n}(\overline{Q}_{ij})_k(h_k^3-h_{k-1}^3)$$

Strains on the reference surface is given by

$$\begin{Bmatrix}\varepsilon_x^o\\ \varepsilon_y^o\\ \gamma_{xy}^o\end{Bmatrix}=\begin{bmatrix}a_{11} & a_{12} & a_{16}\\ a_{12} & a_{22} & a_{26}\\ a_{16} & a_{26} & a_{66}\end{bmatrix}\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\end{Bmatrix} \quad (3)$$

where

$$\begin{bmatrix}a_{11} & a_{12} & a_{16}\\ a_{12} & a_{22} & a_{26}\\ a_{16} & a_{26} & a_{66}\end{bmatrix}=\begin{bmatrix}A_{11} & A_{12} & A_{16}\\ A_{12} & A_{22} & A_{26}\\ A_{16} & A_{26} & A_{66}\end{bmatrix}^{-1}$$

The in-plane elastic constants for a balanced symmetric shaft, with total thickness t are

$$E_x=\frac{1}{t}\left[A_{11}-\frac{A_{12}^2}{A_{22}}\right] \;;\quad E_y=\frac{1}{t}\left[A_{22}-\frac{A_{12}^2}{A_{11}}\right] \;;\quad G_{xy}=\frac{A_{66}}{t} \;;\quad \upsilon_{xy}=\frac{A_{12}}{A_{11}}$$

When a shaft is subjected to torque T, the resultant forces in the laminate by considering the effect of centrifugal forces is

$$N_x = 0 \;; \qquad N_y = 2\rho t r^2 \omega^2 \;; \qquad N_{xy} = \frac{T}{2\pi r^2} \tag{4}$$

The stresses in K th ply are given by

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \qquad \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}_k = \begin{bmatrix} C^2 & S^2 & 2CS \\ S^2 & C^2 & -2CS \\ -CS & CS & C^2 - S^2 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k$$

Knowing the stresses in each ply, the failure of the laminate is determined using the First Ply Failure criteria. That is, the laminate is assumed to fail when the first ply fails. Here maximum stress theory is used to find the torque transmitting capacity

3.4 Torsional Buckling Capacity

Since long thin hollow shafts are vulnerable to torsional buckling, the possibility of the torsional buckling of the composite shaft was checked by the expression for the torsional buckling load T_{cr} of a thin walled orthotropic tube and which is expressed below.

$$T_{cr} = (2\pi r^2 t)(0.272)(E_x E_y{}^3)^{0.25}(t/r)^{1.5} \tag{5}$$

This equation (5) has been generated from the equation of isotropic cylindrical shell and has been used for the design of drive shafts. From this equation, the torsional buckling capability of a composite shaft is strongly dependent on the thickness of composite shaft and the average modulus in the hoop direction.

3.5 Lateral Vibration

Natural frequency based on the Timoshenko beam theory is given by,

$$f_{nt} = K_s \frac{30\pi p^2}{L^2}\sqrt{\frac{E_x r^2}{2\rho}} \qquad \frac{1}{K^2{}_s} = 1 + \frac{p^2 \pi^2 r^2}{2L^2}\left[1 + \frac{f_s E_x}{G_{xy}}\right] \tag{6}$$

The critical speed of the shaft is $N_{crt} = 60 f_{nt}$ (7)

4. DESIGN OPTIMIZATION

Most of the methods used for design optimization assume that the design variables are continuous. In structural optimization, almost all design variables are discrete. A simple Genetic Algorithm (GA) is used to obtain the optimal number of layers, thickness of ply and fiber orientation of each layer. All the design variables are discrete in nature and easily handled by GA. With reference to the middle plane, symmetrical fiber orientations are adopted.

4.1 Objective Function

The objective for the optimum design of the composite drive shaft is the minimization of weight, so the objective function of the problem is given as

Weight of the shaft, $m = \rho A L \;; \quad m = \rho \frac{\pi}{4}\left(d_o^2 - d_i^2\right)L$ (8)

4.2 Design Variables

The design variables of the problem are: 1.Number of plies, 2. Stacking Sequence, and 3. Thickness of the ply and the limiting values of the design variables are given as follows

1]. $n \geq 0$, $n = 1,2,3\ldots32$ 2]. $-90 \leq \theta_k \leq 90$, $k = 1, 2, \ldots\ldots n$ 3]. $0.1 \leq t_k \leq 0.5$

The number of plies required depends on the design constraints, allowable material properties, and thickness of plies and stacking sequence. Based on the investigations it was found that up to 32 numbers of plies are sufficient.

4.3 Design Constraints

1].Torque transmission capacity of the shaft : $T \geq T_{max}$

2.TortioanalBucking capacity of the shaft: $T_{cr} \geq T_{max}$

3. Lateral fundamental natural frequency of the shaft : $N_{crt} \geq N_{max}$

The constraint equations may be written as:

1. $C_1 = \left(1 - \frac{T}{T_{max}}\right)$ If $T < T_{max} = 0$ Otherwise

2. $C_2 = \left(1 - \frac{T_{cr}}{T_{max}}\right)$ If $T_{cr} < T_{max} = 0$ Otherwise

3. $C_3 = \left(1 - \frac{N_{crt}}{N_{max}}\right)$ If $N_{crt} < N_{max} = 0$ Otherwise

$$C = C_1 + C_2 + C_3 \tag{9}$$

Using the method of Rajeev et al[8], the constrained optimization can be converted to unconstrained optimization by modifying the objective function as : $\Phi = m\,(1+k_1C)$ (10)

For all practical purposes, k_1 is a penalty constant and is assumed to be 10.TheInput GA parameters of E-Glass / Epoxy and HM Carbon/Epoxy composite drive shafts of symmetric laminates are shown in the table1. Total string length = String length for number of plies+16*String length for fiber orientation+ String length for thickness of ply =139.

4.4Computer program

A tailor made computer program using C language has been developed to perform the optimization process, and to obtain the best possible design. Fig.3 shown is GA flow chart

4.5 GA Results

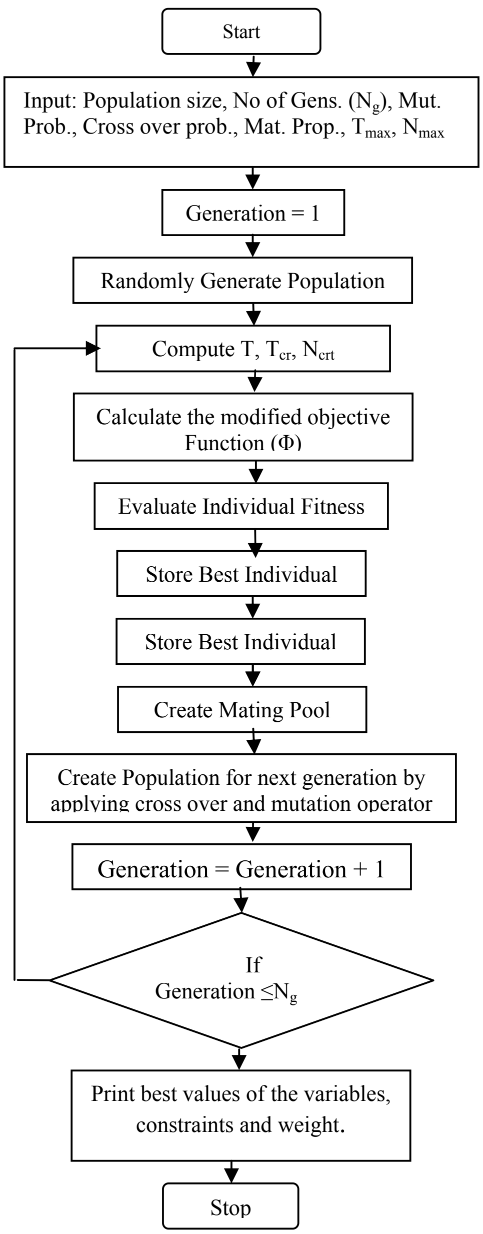

Table 2. Input GA Parameters of composite shafts

GA Parameters	composite drive shaft
Number Of Parameters	:n/2+2,if n is even :(n+1)/2+2,if n is odd
Total string length	:139
Population size	:50
Maximum generations	:150
Cross-over probability	:1
Mutation probability	:0.003
String length for number of plies	:5
String length for fiber orientation	:8
String length for thickness of ply	:6

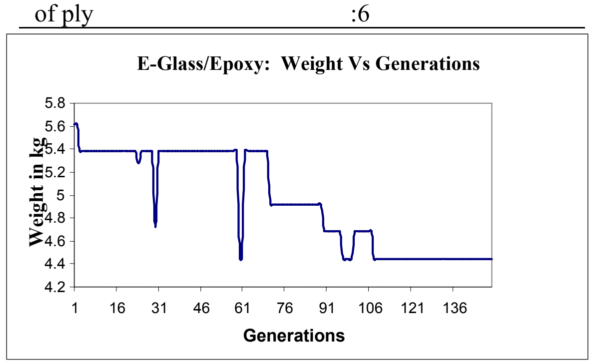

Fig. 4. Variation of No. of Layers of E-Glass/ Epoxy Drive shaft with number of generations

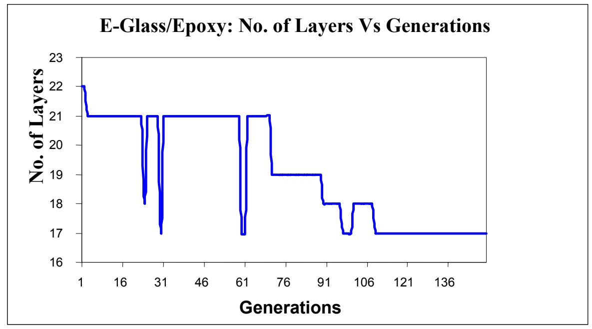

Fig.3.Design flow chart

Fig.5.Variation of No. of Layers of E-Glass/Epoxy Drive shaft with number of generations

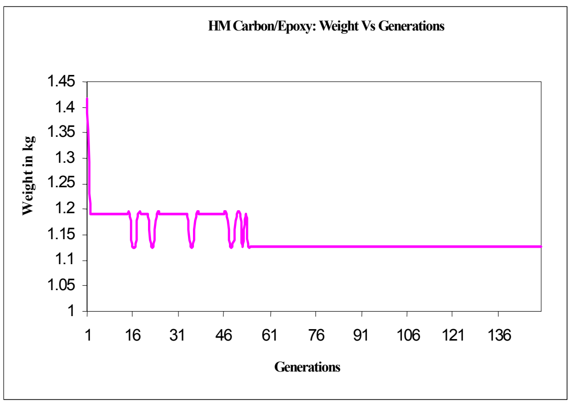

Fig. 6. Variation of the objective function value of HM Carbon/Epoxy shaft with number of generations

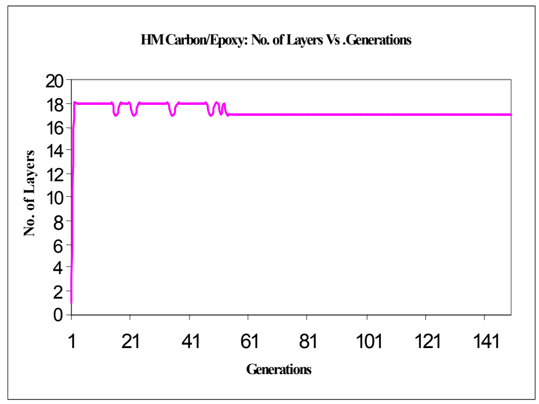

Fig. 7. Variation of Number of Layers of HM Carbon /Epoxy Shaft with number of generations

4.6 Summary of GA results

Table 3 Optimal design values of composite shafts with steel

	d_o (mm)	L (mm)	t_k (mm)	n (plies)	t (mm)	Optimum Stacking sequence	T (Nm)	T_{cr} (Nm)	N_{crt} (rpm)	wt. (kg)	(%) saving
Steel	90	1250	3.32	1	3.32	------	3501	43857	9323	8.6	---
E-Glass/ Epoxy	90	1250	0.4	17	6.8	$[46/-64/-15/-13/39/-84/-28/20/-\overline{27}]_s$	3525	29856	6514	4.4	48.36
HM carbon /Epoxy	90	1250	0.12	17	2.04	$[-65/-25/68/-63/36/-40/-39/74/-\overline{39}]_s$	3656	3765	9270	1.12	86.90

5.0 FINITE ELEMENT ANALYSIS USING ANSYS

5.1 Analysis procedure

In this research, finite element analysis is performed using ANSYS 5.4 software. To model both the composite shaft, the shell 99 element is used and the shaft is subjected to torsion. The shaft is fixed at one end in axial, radial and tangential directions and is subjected to torsion at the other end. After performing a static analysis of the shaft, the stresses are saved in a file to calculate the buckling load. The output of the buckling analysis is a load coefficient which is the ratio of the buckling load to the static load. This software also calculates the modes of buckling of the composite shaft. The analysis results obtained for E-Glass/epoxy and HM carbon/epoxy composite shafts are for optimal stacking sequences took from GA. For Critical speed analysis, the boundary condition considered as pinned pinned condition. The modal analysis is performed to find the natural frequencies in lateral directions.The frequencies obtained are then multiplied by 60 to obtain critical speeds as material natural frequencies. The mode shapes for all material combinations are obtained to their corresponding critical speeds.

5.2 Finite element analysis to calculate torsional buckling load of composite shaft

In Figs. 8 and 9, the mesh configuration and the first mode of buckling of the E-glass/epoxy and Hm-Carbon/eoxy shafts composite shaft are shown. In Table 1, the results of the buckling torque obtained from closed form solution are shown. The results obtained from Finite element analysis show good agreement with GA results the ply sequence has an important effect on the torsional buckling of the shaft

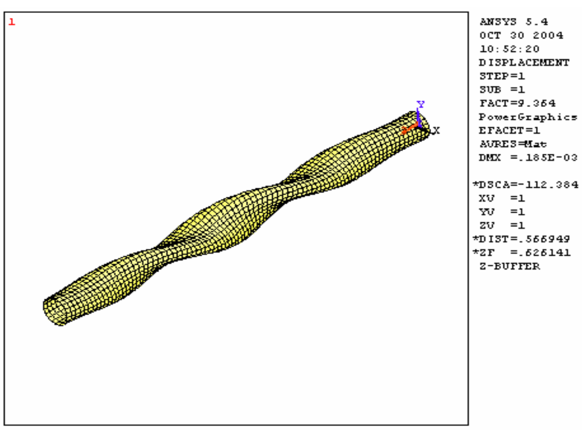

Fig. 8. First mode of torsional buckling of E-glass epoxy composite shaft

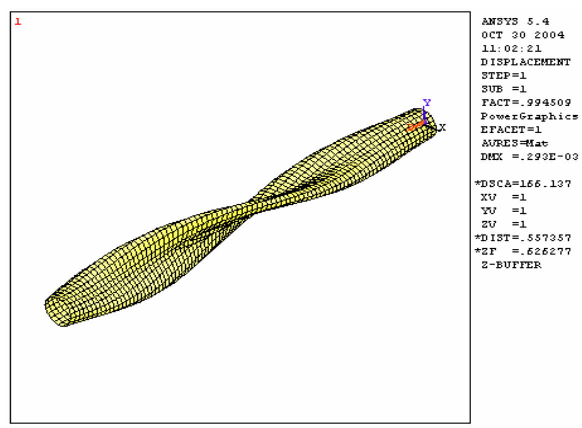

Fig. 9 First mode of torsional buckling of HM carbon composite shaft

5.3 Variation of torsional frequency of a composite shaft due to applied torque

In Figs. 10 and 11, the mesh configuration to analyze whirling of E-glass/epoxy and Hm-Carbon/eoxy shafts composite shaft is shown. In Table 4, the results of critical speed obtained from closed form solution are shown. Finite element analysis and GA results which shows increasing the applied torque decreases the natural frequencies of torsion and does not change other modes.

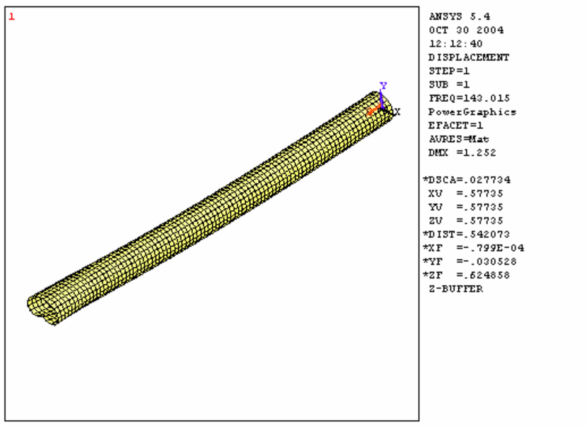

Fig. 10. critical speed of E-Glass/ glass epoxy composite shaft[first mode]

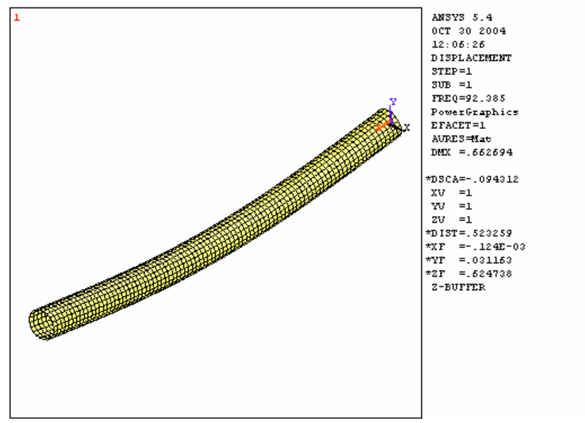

Fig.11. critical speed of HM Carbon composite shaft [first mode]

Table 4 Comparison between finite element and GA methods

		Steel	E-glass/ epoxy	HM carbon/ epoxy
Optimal stacking sequence from GA		------	$[46/-64/-15/-13/39/-84/-28/20/-\overline{27}]_s$	$[-65/25/68/-63/36/-40/-39/74/-\overline{39}]_S$
			Eigen Buckling analysis (Nm)	
Critical buckling torque T_{cr} (N.m):	GA	43857.96	29856.45	3765.75
Buckling load factor :	ANSYS	13.835	9.364	0.9945
Critical buckling torque T_{cr} (N.m) = Buck.factor * applied torque from GA:	ANSYS	48447.68	33010	3636.56
	% Deviation	10.46	10.56	3.56
			Critical speed(rpm)	
	GA	9323.68	6514.56	9270.3
	ANSYS	9385.8	5543.1	8580.6
	% Deviation	0.66	14.91	7.43

6.0. CONCLUDING REMARKS

- A procedure to design a composite drive shaft is suggested.
- Drive shaft made up of E-Glass/ Epoxy and HS Carbon/Epoxy multilayered composites has been designed.
- The designed drive shafts are optimized using GA and analyzed using ANSYS for better stacking sequence, better torque transmission capacity and bending vibration characteristics.
- The usage of composite materials and optimization techniques has resulted in considerable amount of weight saving in the range of 48 to 86% when compared to steel shaft.
- The fiber orientation of a composite shaft strongly affects the buckling torque
- The finite element modeling presented in this analysis is able to predict the buckling torque.
- These results are encouraging and suggest that GA can be used effectively and efficiently in other complex and realistic designs often encountered in engineering applications

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Notation

A_{ij} : Extensional stiffness matrix
a_{ij} : Inverse of the Extensional stiffness matrix
B_{ij} : Coupling stiffness matrix
d_i & d_o : Inner diameter of the shaft
D_{ij} : Bending stiffness matrix
S & C : Sinθ and Cosθ
E_{11} & E_{22} :Long.& trans. elastic modulus of lamina
E_x & E_y : Elastic modulus of the shaft in axial(X) & transverse. (Y) direction
f_s : Shape factor(=2 for hollow circular sections)
f_{nt} : Natural Frequency based on Timoshenko beam theory
G : Shear Modulus, GPa
G_{12} : Shear modulus of lamina in 12-dirn.
G_{xy} : Shear modulus of the shaft in XY-dirn.
h_k : Dist. bt. the neutral fiber to the top of K^{th} layer
i,j :1,2,6
k : Ply number,
K_s : Shear coefficient of the lat. natural frequency
L : Length of the shaft
m : Weight of the shaft
n : Total Number of plies
N_g : Number of generations
N_{max} :Maximum speed of the shaft
N_{crt} : Critical Speed of the shaft based on Timoshenko theory
N_x, N_y and Nxy: Normal force/unit length in X,Y and shear force/ unit length XY-dirn.,
p :1, 2,3. (1= First natural frequency)
Q_{ij} & $\overline{Q}_{ij}$:stiffness& transformed stiffness matrices
r :Mean radius of the shaft
S_s : Shear Strength
S_y : Yield Strength
S^t_1 & S^c_1 : long. tensile & compressive strength,
S^t_2 & S^C_2 : trans. tensile& compressive strength,
S_{12} :Ultimate in-plane shear strength
t :Thickness of shaft
t_k : ply thickness
T : Torque transmission capacity of the shaft
T_{max} : Ultimate torque
T_{cr} : Torsional buckling capacity of the shaft
V_f : Fiber volume fraction
ν : Poisson's ratio
ν_{12} : Major Poisson's ratio
ρ : Density of the shaft material
θ : Fiber orientation angle, degrees
$\varepsilon_1, \varepsilon_2$ & γ_{12} :Normal strain in longitudinal, transverse and shear strain in 12- direction
ε_x , ε_y & γ_{xy} :Normal strain in X,Y- direction and Shear strain in XY- direction
κ^o_x κ^o_y & κ^o_{xy} :Midplane curvature in X,Y- dirn./m and Midplane twisting curvature in XY-direction/m
$\varepsilon^o_x, \varepsilon^o_y$ and γ^o_{xy} :Midplane extensional strain in X,Y direction and shear strain in XY-dirn.
σ_1, σ_2 and τ_{12} :Normal Stress acting in the long. and transverse dirn. and shear Stress acting in 12-direction of a lamina
σ_x, σ_y and τ_{xy} : Normal Stresses acting along X,Y and Shear Stress acting in XY dirn. of a lamina,
ω : Angular velocity