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GENETIC ALGORITHM BASED OPTIMUM DESIGN OF COMPOSITE DRIVE SHAFT

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ABSTRACT

In this work, an attempt has been made to optimize design parameters of a composite drive shaft, which replaces a conventional steel shaft in an automobile power-train, using Genetic Algorithm (GA). The parameters such as ply thickness, number of plies and stacking sequence were optimized for E-Glass/Epoxy and Boron/Epoxy shafts using GA with the objective of weight minimization of the composite shaft which is subjected to constraints such as torque transmission, torsional buckling load and fundamental natural frequency. The weight reduction can be achieved considerably. The stresses distributed along shaft thickness were analyzed and found to be within allowable limits.

Keywords: Stacking sequence; Genetic algorithms; Optimization; composite drive shaft; weight reduction

1. INTRODUCTION

Substituting composite structures for conventional metallic structures has many advantages because of higher specific stiffness and strength of composite materials. Advanced composite materials seem ideally suited for long, power drive shaft applications. Their elastic properties can be tailored to increase the torque and the rotational speed at which they operate. The advanced composite materials such as Boron, Graphite, Carbon, Kevlar and Glass with suitable resins are widely used because of their high specific strength (strength/density) and high specific modulus (modulus/density)[1]. Polymer matrix composites were proposed for light weight shafts in drivelines for automotive[3, 4] industries. A GA based on natural genetics has been used for this work[5]. The fairly new GA was and applied for the design optimization of steel and composite leaf springs in the previous study by authors[6, 7]. Most of the automobiles employs shafts in drive-trains and weight reduction of drive shaft by optimization of design parameters is highly desirable if it can be achieved without cost increase and loss of quality and reliability.

In the present work it has been attempted to evaluate the use of E-Glass/Epoxy and Boron/Epoxy composites for automotive drive shafts and a single piece composite drive shaft for rear wheel drive automobile was optimally designed with composites using GA with weight reduction as the objective and keeping torque transmission, torsional buckling strength capabilities and natural bending frequency as constraints.

2. DESIGN OBJECTIVES

The torque transmission capability of the drive shaft for passenger cars, small trucks, and vans should be larger than 3,500 Nm and fundamental natural bending frequency of the shaft should be higher than 6,500 rpm to avoid whirling vibration. The outer diameter (do) restricted to100 mm due to space limitations and here it is taken as 90 mm. The drive shaft was designed optimally to the specified design requirements[5].

3. DESIGN OF COMPOSITE DRIVE SHAFT

3.1. Assumptions

The shaft has a uniform, circular cross section and rotates at a constant speed about its longitudinal axis. The shaft is perfectly balanced, i.e., at every cross section, the mass center coincides with the geometric center. All damping and nonlinear effects are excluded. The stress-strain relationship for composite material is linear & elastic; hence, Hook's law is applicable for composite materials. Since lamina is thin and no out-of-plane loads are applied, it is considered as under the plane stress

3.2. Selection of Cross-Section and Materials

The E-Glass/Epoxy and Boron/Epoxy composites are selected for drive shaft. Since, composites are highly orthotropic and their fractures were not fully studied. The factor of safety was taken as 2 and the fiber volume fraction as 0.6.

3.3. Torque transmission capacity of the composite drive shaft

3.3.1. Stress-Strain Relationship for Unidirectional Lamina

Since the lamina is thin and no out-of-plane loads are applied, it is considered as the plane stress problem and 3-D problem can be reduced into 2-D problem. For unidirectional 2-D lamina, the stress-strain relation ship in terms of physical material direction is given by

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix} \qquad (1)$$

The matrix Q is referred as the reduced stiffness matrix for the layer and its terms are given by

$$Q_{11}=\frac{E_{11}}{1-\nu_{12}\nu_{21}}\ ; \qquad Q_{12}=\frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}}\ ;$$

$$Q_{22}=\frac{E_{22}}{1-\nu_{12}\nu_{21}}\ ; \qquad Q_{66}=G_{12}$$

For an angle-ply lamina, where fibers are oriented at an angle with the positive X-axis (Longitudinal axis of shaft), the stress strain relationship is given by,

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}\overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}}\\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}}\\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} \qquad (2)$$

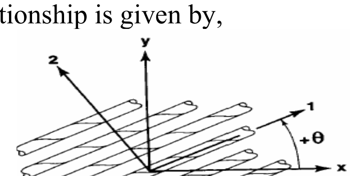

Fig. 2. principal materials axes from x-y axes

$$\overline{Q_{11}}=Q_{11}c^4+Q_{22}s^4+2(Q_{12}+2Q_{66})s^2c^2$$

$$\overline{Q_{12}}=(Q_{11}+Q_{22}-4Q_{66})s^2c^2+Q_{12}(c^4+s^4)$$

$$\overline{Q_{16}}=(Q_{11}-Q_{12}-2Q_{66})c^3s-(Q_{22}-Q_{12}-2Q_{66})cs^3$$

For a symmetric laminate the force and moment resultants are

$$\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\end{Bmatrix}=\begin{bmatrix}A_{11} & A_{12} & A_{16}\\ A_{12} & A_{22} & A_{26}\\ A_{16} & A_{26} & A_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x^o\\ \varepsilon_y^o\\ \gamma_{xy}^o\end{Bmatrix} \qquad (3)$$

$$\overline{Q_{22}} = Q_{11}S^4 + Q_{22}S^4 + 2(Q_{11} + 2Q_{66})S^2C^2$$

$$\overline{Q_{26}} = (Q_{11} - Q_{12} - 2Q_{66})CS^3 - (Q_{22} - Q_{12} - 2Q_{66})C^3S$$

$$\overline{Q_{66}} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}(s^4 + c^4)$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x^o \\ \kappa_y^o \\ \kappa_{xy}^o \end{Bmatrix} \quad (4)$$

where S= sin Θ, C= cos Θ
For a symmetric laminate, the B matrix vanishes and the in plane and bending stiff-nesses are uncoupled.

$$A_{ij} = \sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k - h_{k-1}); \qquad B_{ij} = \frac{1}{2}\sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^2 - h_{k-1}^2); \qquad D_{ij} = \frac{1}{3}\sum_{k=1}^{n} (\overline{Q_{ij}})_k (h_k^3 - h_{k-1}^3)$$

Strains on the reference surface is given by where

$$\begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} \quad (5) \qquad \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}^{-1}$$

The in-plane elastic constants for a balanced symmetric shaft, with total thickness t are

$$E_x = \frac{1}{t}\left[A_{11} - \frac{A_{12}^2}{A_{22}}\right]; \quad E_y = \frac{1}{t}\left[A_{22} - \frac{A_{12}^2}{A_{11}}\right]; \quad G_{xy} = \frac{A_{66}}{t}; \quad \upsilon_{xy} = \frac{A_{12}}{A_{11}}$$

When a shaft is subjected to torque T, the resultant forces in the laminate by considering the effect of centrifugal forces is

$$N_x = 0 \qquad N_y = 2\rho t r^2 \omega^2 \qquad N_{xy} = \frac{T}{2\pi r^2} \quad (6)$$

The stresses in K th ply are given by

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{12}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \qquad \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}_k = \begin{bmatrix} C^2 & S^2 & 2CS \\ S^2 & C^2 & -2CS \\ -CS & CS & C^2 - S^2 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k$$

After evaluating the stresses in each ply, the failure of the laminate is determined using the First Ply Failure criteria. That is, the laminate is assumed to fail when the first ply fails. Here maximum stress theory is used to find the torque transmitting capacity

3.4. Torsional Buckling Load

Since long thin hollow shafts are vulnerable to torsional buckling, the possibility of the torsional buckling of the composite shaft was checked by the expression for the torsional buckling load T_{cr} of a thin walled orthotropic tube and is expressed below.

$$T_{cr} = (2\pi r^2 t)(0.272)(E_x E_y^3)^{0.25}(t/r)^{1.5}$$

(7)

This equation (7) has been generated from the equation of isotropic cylindrical shell and has been used for the design of drive shafts. From this equation, it is seen that the torsional buckling capability of a composite shaft is strongly dependent on the thickness of composite shaft and the average modulus in the hoop direction.

3.5. Whirling frequency

Natural frequency based on the Timoshenko beam theory is given by,

$$f_{nt} = K_s \frac{30\pi p^2}{L^2}\sqrt{\frac{E_x r^2}{2\rho}} \quad ; \quad \frac{1}{K^2_s} = 1 + \frac{p^2\pi^2 r^2}{2L^2}\left[1 + \frac{f_s E_x}{G_{xy}}\right] \tag{8}$$

The critical speed of the shaft is $N_{crt} = 60 f_{nt}$ (9)

4. DESIGN OPTIMIZATION

Most of the design optimization methods assume that the design variables are continuous. In structural optimization, almost all design variables are discrete. A simple Genetic Algorithm (GA) is used to obtain the optimal number of layers, thickness of ply and fiber orientation of each layer. All the design variables are discrete in nature and easily handled by GA. With reference to the middle plane, symmetrical fiber orientations are adopted.

4.1. Objective Function

The objective for the optimum design of the composite drive shaft is the minimization of weight, so the objective function of the problem is given as

Weight of the shaft, $m = \rho A L$; $m = \rho \frac{\pi}{4}\left(d_o^2 - d_i^2\right)L$ (10)

4.2. Design Variables

The design variables of the problem are: 1.Number of plies, 2. Stacking Sequence, and 3. Thickness of the ply and the limiting values of the design variables are given as follows

1]. $n \geq 0$ 2]. $-90 \leq \theta_k \leq 90$ 3]. $0.1 \leq t_k \leq 0.5$

n = 1,2,3…32 k =1, 2,……

n

The number of plies required depends on the design constraints, allowable material properties, thickness of plies and stacking sequence. Based on the investigations it was found that up to 32 numbers of plies are sufficient.

4.3. Design Constraints

1].Torque transmission capacity of the shaft : $T \geq T_{max}$

2].TortioanalBucking capacity of the shaft: $T_{cr} \geq T_{max}$

3]. Lateral fundamental natural frequency of the shaft : $N_{crt} \geq N_{max}$

The constraint equations may be written as:

1]. $C_1 = \left(1 - \frac{T}{T_{max}}\right)$ If $T < T_{max}$ = 0 Otherwise

2]. $C_2 = \left(1 - \frac{T_{cr}}{T_{max}}\right)$ If $T_{cr} < T_{max}$ = 0 Otherwise

3]. $C_3 = \left(1 - \frac{N_{crt}}{N_{max}}\right)$ If $N_{crt} < N_{max}$ = 0 Otherwise

$$C = C_1 + C_2 + C_3 \quad (11)$$

Using the method of Rajeev et al[8], the constrained optimization can be converted to unconstrained optimization by modifying the objective function as :

$$\Phi = m\,(1+k_1C) \quad (12)$$

For all practical purposes, k_1 is a penalty constant and is assumed to be 10. The Input GA parameters of E-Glass / Epoxy and Boron/Epoxy composite drive shafts of symmetric laminates are shown in the table1. Total string length = String length for number of plies+16*String length for fiber orientation+ String length for thickness of ply =139.

5. COMPUTER PROGRAM

A tailor made computer program using C language has been developed to perform the optimization process, and to obtain the best possible design. Fig.3 shown is GA flow chart

6.RESULTS AND DISCUSSION

6.1.1. GA Results for E-Glass/Epoxy Shaft shown in figure 4 and figure 5

6.1.2. GA Results for Boron/Epoxy Shaft shown in figure 6 and figure 7

7. CONCLUDING REMARKS

- A procedure to design a composite drive shaft is suggested**.**
- Drive shaft made up of E-Glass/ Epoxy and Boron/Epoxy multilayered composites have been designed.
- The designed drive shafts are optimized using GA for better stacking sequence, better torque transmission capacity and bending vibration characteristics.
- The usage of composite materials and optimization techniques has resulted in considerable amount of weight saving in the range of 48% to 86% when compared to steel shaft.
- These results are encouraging and suggest that GA can be used effectively and efficiently in other complex and realistic designs often encountered in engineering applications.
- The stresses and strains along the thickness of the shaft are found to be within allowable limit.

Notation

A_{ij} : Extensional stiffness matrix
a_{ij} : Inverse of the Extensional stiffness matrix
B_{ij} : Coupling stiffness matrix
d_i & d_o : Inner diameter of the shaft
D_{ij} : Bending stiffness matrix
S & C : Sinθ and Cosθ
E_{11} & E_{22} :Long.& trans. elastic modulus of lamina
E_x & E_y : Elastic modulus of the shaft in axial(X) & transverse. (Y) direction
f_s : Shape factor(=2 for hollow circular sections)
f_{nt} : Natural Frequency based on Timoshenko beam theory
G : Shear Modulus, GPa
G_{12} : Shear modulus of lamina in 12-dirn.
G_{xy} : Shear modulus of the shaft in XY-dirn.
h_k : Dist. bt. the neutral fiber to the top of K^{th} layer
i,j :1,2,6
k : Ply number,
K_s : Shear coefficient of the lat. natural frequency
L : Length of the shaft
m : Weight of the shaft
n : Total Number of plies
N_g : Number of generations
N_{max} :Maximum speed of the shaft
N_{crt} : Critical Speed of the shaft based on Timoshenko theory
N_x, N_y and Nxy: Normal force/unit length in X,Y and shear force/ unit length XY-dirn.,
p :1, 2,3. (1= First natural frequency)
Q_{ij} & $\overline{Q}_{ij}$:stiffness& transformed stiffness matrices
r :Mean radius of the shaft
S_s : Shear Strength
S_y : Yield Strength
S^t_1 & S^c_1 : long. tensile & compressive strength,
S^t_2 & S^C_2 : trans. tensile& compressive strength,
S_{12} :Ultimate in-plane shear strength
t :Thickness of shaft
t_k : ply thickness
T : Torque transmission capacity of the shaft
T_{max} : Ultimate torque
T_{cr} : Torsional buckling capacity of the shaft
V_f : Fiber volume fraction
ν : Poisson's ratio
ν_{12} : Major Poisson's ratio
ρ : Density of the shaft material
θ : Fiber orientation angle, degrees
$\varepsilon_1, \varepsilon_2 \,\&\, \gamma_{12}$: Normal strain in longitudinal, transverse and shear strain in 12-direction
ε_x , $\varepsilon_y \,\&\, \gamma_{xy}$: Normal strain in X,Y- direction and Shear strain in XY- direction
$\kappa^o_x\ \kappa^o_y$ & κ^o_{xy} : Midplane curvature in X,Y-dirn./m and Midplane twisting curvature in XY-direction/m
$\varepsilon^o_x, \varepsilon^o_y$ and γ^o_{xy} :Midplane extensional strain in X,Y direction and shear strain in XY-dirn.
σ_1, σ_2 and τ_{12} :Normal Stress acting in the long. and transverse dirn. and shear Stress acting in 12-direction of a lamina
σ_x, σ_y and τ_{xy} : Normal Stresses acting along X,Y and Shear Stress acting in XY dirn. of a lamina,
ω : Angular velocity

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TABLES

Table 1. Input GA Parameters of composite shafts

GA Parameters	composite drive shaft
Number Of Parameters	:n/2+2,if n is even :(n+1)/2+2,if n is odd
Total string length	:139
Population size	:50
Maximum generations	:150
Cross-over probability	:1
Mutation probability	:0.003
String length for number of plies	:5
String length for fiber orientation	:8
String length for thickness of ply	:6

Table 2. Mechanical properties for each lamina of the laminate

	E-Glass/ Epoxy	Boron/ Epoxy
E_{11} (GPa)	50.0	204.0
E_{22} (GPa)	12.0	18.5
G_{12} (GPa)	5.6	5.59
ν_{12}	0.3	0.23
$\sigma^T_1 = \sigma^C_1$ (MPa)	800.0	1260.0
$\sigma^T_2 = \sigma^C_2$ (MPa)	40.0	61.0
τ_{12} (MPa)	72.0	67.0
ρ (Kg/m^3)	2000.0	2000.0

Table 3 comparison of allowable and predicted stresses and strains along the length of shaft

Material	Allowable Stress(MPa)	Predicted Stress (MPa)	Design is OK/NOT
E-Glass/Epoxy	$S^t_1 = 400$	158	OK
	$S^t_2 = 20$	9.3	OK
	$S_{12} = 36$	6.4	OK
	$S^C_1 = -400$	- 170	OK
	$S^C_2 = -20$	-6.2	OK
	$S_{12} = -36$	-7.7	OK
Boron/Epoxy	$S^t_1 = 630$	279	OK
	$S^t_2 = 30$	23.2	OK
	$S_{12} = 30$	6.5	OK
	$S^C_1 = -630$	- 234	OK
	$S^C_2 = -30$	-23.4	OK
	$S_{12} = -30$	-6.6	OK

Table 4 Optimal design values of composite shafts with steel

	d_o (mm)	L (mm)	t_k (mm)	n (plies)	t (mm)	Optimum Stacking sequence	T (Nm)	T_{cr} (Nm)	N_{crt} (rpm)	wt. (kg)	(%) saving
Steel	90	1250	3.32	1	3.32	------	3501	43857	9323	8.6	---
E-Glass/ Epoxy	90	1250	0.4	17	6.8	$[46/-64/-15/-13/39/-84/-28/20/\overline{-27}]_S$	3525	29856	6514	4.4	48.36
Boron /Epoxy	90	1250	0.2	9	1.8	$[-66/62/-27/13/\overline{67}]_s$	3558	3791	11089	1.24	85.58

FIGURES

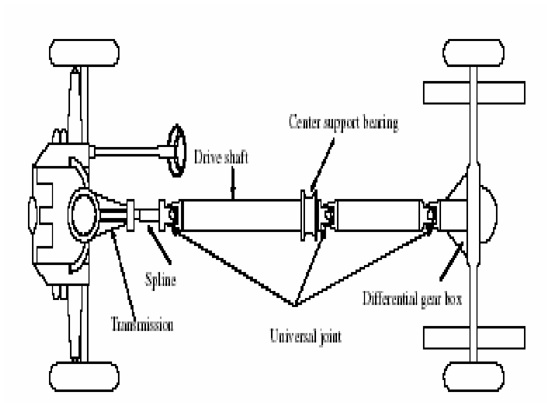


Fig.1. Conventional two-piece drive shaft arrangement for rear wheel vehicle driving system [10]

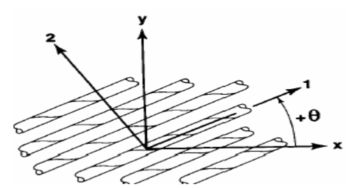


Fig. 2. principal materials axes from x-y axes

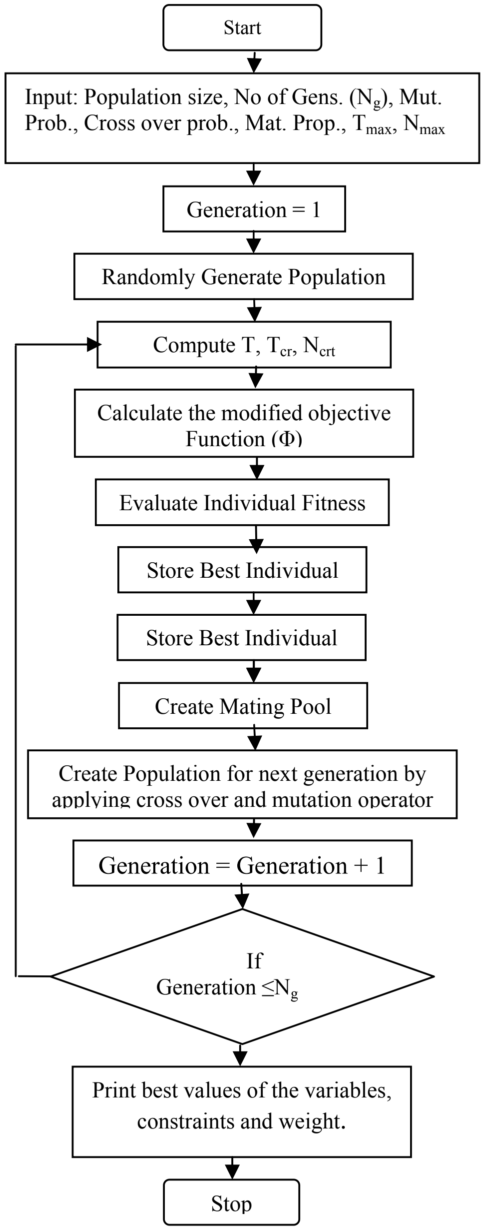
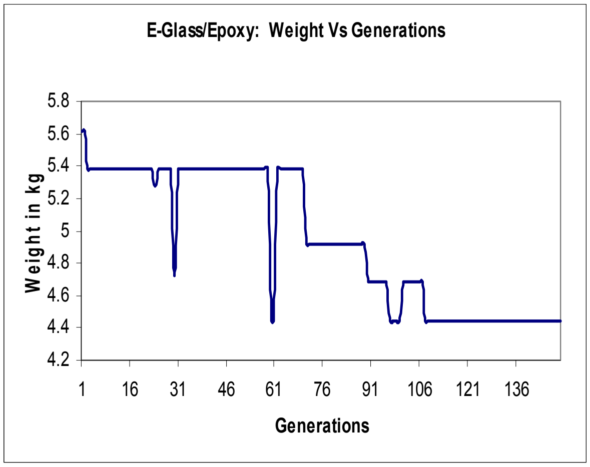


Fig.3. Design flow chart



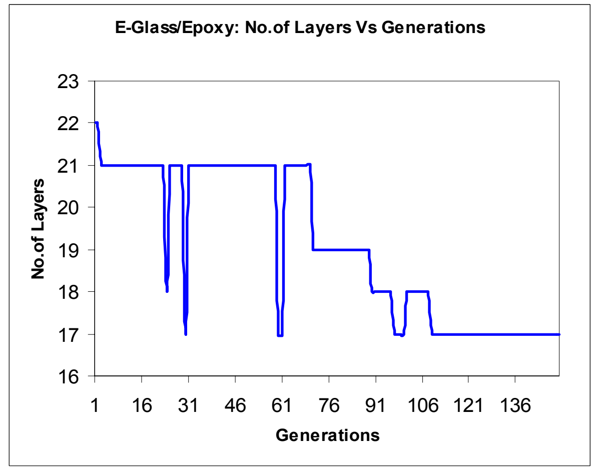
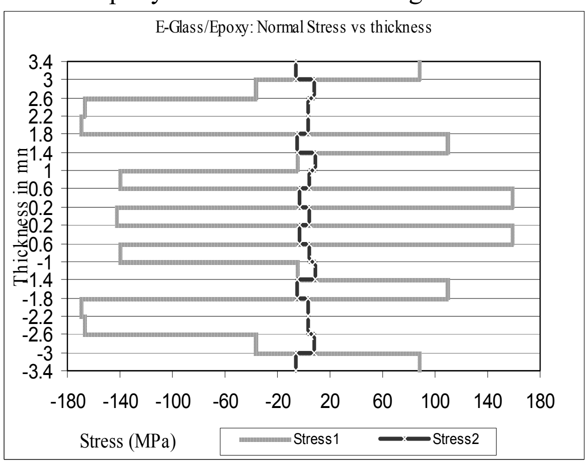


Fig. 4. Variation of the objective function value of E-Glass/Epoxy shaft with number of generations

Fig. 5. Variation of Number of Layers of E-Glass /Epoxy Shaft with number of generations



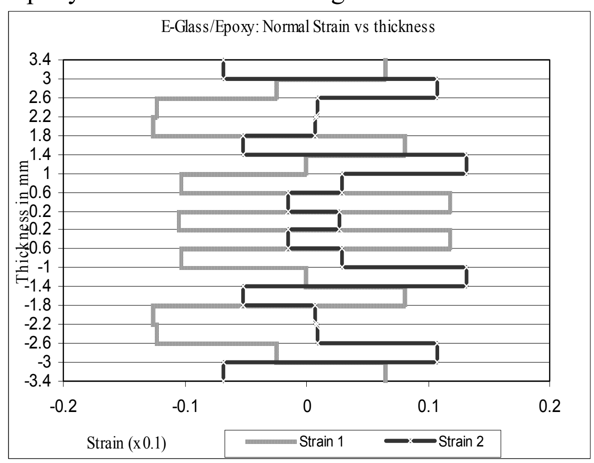
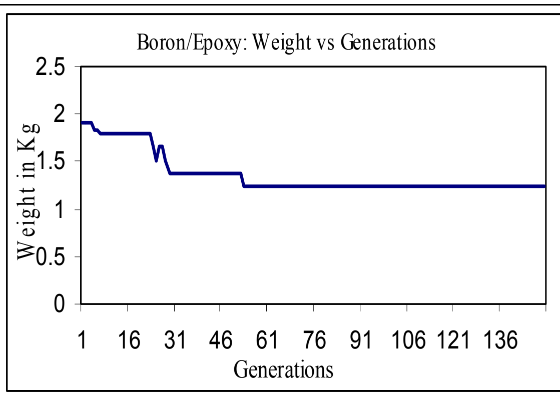


Fig. 6. Variation of normal stress along the thickness of E-Glass/Epoxy shaft with number of generations

Fig. 7. Variation of normal strain along the thickness of E-Glass/Epoxy shaft with number of generations



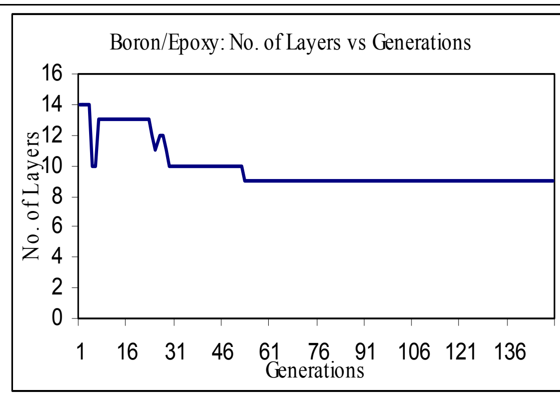


Fig. 8. Variation of the objective function value of Boron/Epoxy shaft with number of generations

Fig. 9. Variation of Number of Layers of Boron /Epoxy Shaft with number of generations

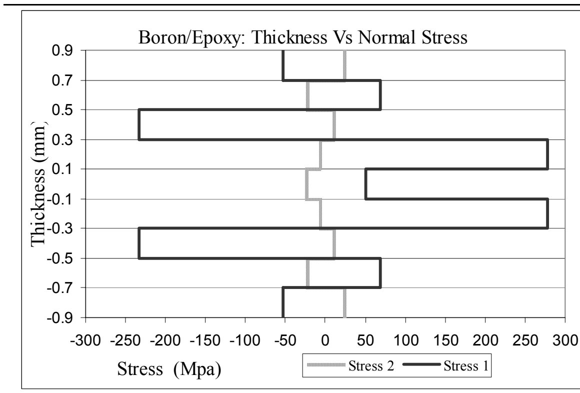


Fig. 10. Variation of normal stress along the thickness of Boron/Epoxy shaft with number of generations

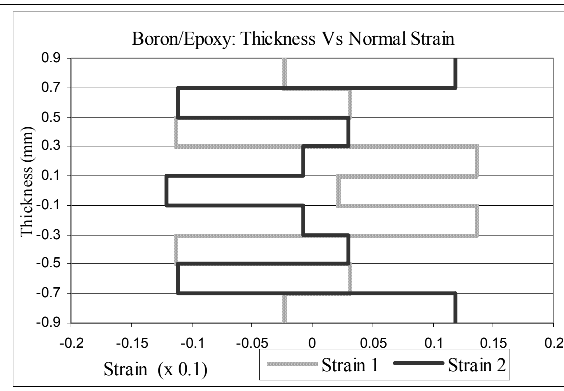


Fig. 11. Variation of normal strain along the thickness of Boron/Epoxy shaft with number of generations