NEW BAINITIC STEELS BY DESIGN

Displacive Phase Transformations and their Applications in Materials Engineering, eds K. Inoue, K. Mukherjee, K. Otsuka and H. Chen, The Minerals, Metals and Materials Society, Warrendale, Pennsylvania, U.S.A., (1998), pp. 69–78

H. K. D. H. Bhadeshia
University of Cambridge
www.msm.cam.ac.uk/phase-trans

Abstract

Bainitic steels are now at the forefront of new and exciting developments in steel technology. This paper discusses how phase transformation theory is being exploited in the systematic design of every aspect of the new alloys, emphasising the immense influence that Professor Wayman has had on the development of the subject [1].

Technology of Bainitic Steels

Gone are the days when industry could simply make a whole matrix of alloys, test them, and then hope for the best. Such methods are less acceptable in the modern competitive environment. This realisation has caused a great increase in research leading to the development of models which help reduce the time and expense of alloy development, and maintain the enormous success of steel as a structural material.

It is therefore fortunate that bainitic steels have become popular at a time when it is possible to systematically design them using a panoply of quantitative techniques based on phase transformation theory. The purpose of this paper is to show how such theory has led already to the development of important new alloys. The rise in popularity of bainitic steels is for a number of reasons. Modern processing techniques (such as accelerated cooling mills) have enabled steels which would traditionally be considered as having poor hardenability, to be produced with homogeneously bainitic microstructures. Consequently, coarse carbides are avoided and the ultrafine, submicrometer grain size of the bainitic ferrite can be exploited to give a strong and tough steel with weldability. The inevitable solidification–induced chemical segregation present in commercial steels leads to a "banding" when the microstructure is a mixture of allotriomorphic ferrite and pearlite. This is not the case when the transformation occurs at a larger supersaturation to bainite, giving major advantages in corrosive environments. We shall begin this review by discussing aspects of the mechanism which are necessary in order to calculate the structure of bainite, and then illustrate how the mechanism can be exploited in design. Table 1 lists some of the properties which are of interest in industrial design, and which need to be related directly to phase transformation theory. Only a few of these relations are discussed here; more can be found elsewhere [2].

Property	Relevance
Yield strength	All structural applications
Ultimate tensile strength	All structural applications
YS/UTS ratio	Tolerance to plastic overload
Elongation	Resistance to brittle fracture
Uniform elongation	Related to YS and UTS
Non–uniform elongation	Related to inclusions
Toughness	Tolerance to defects
Fatigue	Cyclic loading, life assessments
Stress corrosion	Slow corrosion & cracking
Creep strength	High temperature service
Creep ductility	Safe design
Creep–fatigue	Fatigue at creep temperatures
Elastic modulus	Deflection, stored energy
Thermal expansivity	Thermal fatigue
Hardness	Tribological properties

Table 1: Mechanical properties that need to be related to transformation theory.

Pure Iron

Pure iron can be transformed into equiaxed–grains of ferrite or into martensite, depending on the rate at which it is quenched from the austenite phase field. In the first case the atoms of the austenite are redistributed by diffusion into the new arrangement of ferrite, with the flow of atoms occurring in a way which minimises the strain of transformation [3]. The luxury of atomic flow is not available during martensitic transformation, where the crystal structure is changed by a homogeneous deformation of the austenite (the Bain Strain [4]), a mechanism which is *displacive.* Any deformation will lead to an observable change in shape; Wayman and Alstetter in 1962 demonstrated

.

the large displacements associated with the formation of martensite in zone–refined iron [5].

It follows from this discussion that (a) diffusion is necessary during reconstructive transformation irrespective of whether there is any redistribution of solute; (b) the shape of the transformed region will change when the transformation of structure is achieved by deformation.

Mechanism of Displacive Transformation

The Bain strain converts the structure of the parent phase into that of the product phase. When combined with an appropriate rigid body rotation, the net homogeneous lattice deformation **RB** is an invariant–line strain (Fig. 1), giving good fit only along a single line common to the two crystals. However, the observed shape deformation indicates a larger degree of fit. The shape change is in fact an invariant–plane strain **P**, but this gives the wrong crystal structure. If, however, a second homogeneous shear **Q** is combined with **P**, then the correct structure is obtained but the wrong shape since

$$\mathbf{PQ} = \mathbf{RB}$$

These discrepancies are all resolved if the shape changing effect of **Q** is canceled macroscopically by an inhomogeneous lattice–invariant deformation, which may be slip or twinning as illustrated in Fig. 1.

The theory illustrated explains all the observed features of the martensite crystallography. It is easy to predict the orientation relationship, by deducing the Bain strain and adding a rigid body rotation which makes the net lattice deformation an invariant–line strain. The habit plane does not have rational indices because the amount of lattice–invariant deformation needed to recover the correct macroscopic shape is not usually rational. The theory predicts a substructure in plates of martensite (either twins or slip steps) as is observed experimentally. The transformation goes to all the trouble of ensuring that the shape deformation is macroscopically an invariant–plane strain because this reduces the strain energy when compared with the case where the shape deformation might be an invariant–line strain.

We now discuss a number of the consequences of the mechanism of transformation, consequences which make bainite so useful in practice.

Consequences: The Shape

The equilibrium shape of ferrite in austenite is that of an equiaxed idiomorph. Many varieties of ferrite nevertheless grow in the form of plates. These all grow by the displacive mechanism described above, so that their growth causes the shape of the transformed region to change, the change being an invariant–plane strain (IPS) with a large shear (Table 2, [7–11]). This is not the case when the ferrite grows in any other shape. A small aspect ratio (c/r, Fig. 2) leads to a better accommodation of the plate and hence to a minimisation of the elastic strains in the austenite.

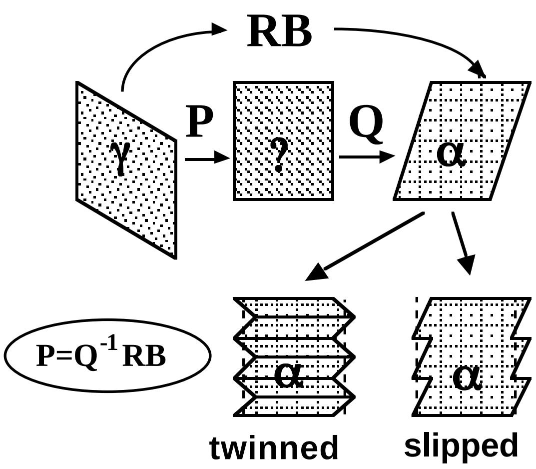

Fig. 1: Phenomenological theory of martensite crystallography [6]. γ and α refer to austenite and martensite respectively.

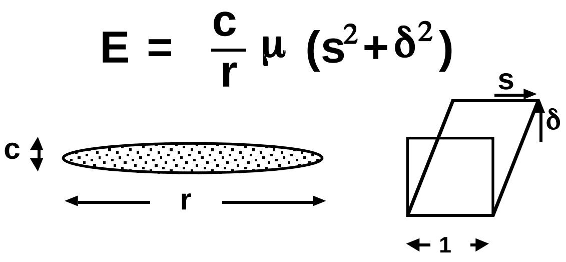

Fig. 2: Strain energy per unit volume, E, when the IPS shape deformation is elastically accommodated. μ is the shear modulus of the austenite. The diagram on the right shows how the shape of a unit–cube changes due to the IPS deformation, defined by the dilatational and shear strains δ and s respectively.

Processes of plastic deformation absorb a lot more energy than when cleavage occurs. Therefore, most mechanisms of strengthening lead to a decrease in toughness. The exception is grain size strengthening, where a refinement of the microstructure simultaneously improves the strength and toughness [12]. In

Transformation	s	δ	Morphology	Ref.
Widmanstätten ferrite	0.36	0.03	Thin plates	[7]
Bainite	0.22	0.03	Thin plates	[8,9]
Martensite	0.24	0.03	Thin plates	[10,11]
Allotriomorphic α	0	0.03	irregular	
Idiomorphic α	0	0.03	equiaxed	

Table 2: Approximate values of s and δ for a variety of transformation products in steels.

this respect, a plate shape is better than an equiaxed grain structure. The mean free slip distance through a plate is only about twice its thickness [13], which for bainite is typically far less than a micrometer in size. The accelerated cooled thermomechanically processed steels with bainitic microstructures rely on this.

Consequences: Nature of Interface

A planar interface traversing a crystal and having no long–range elastic field produces an invariant–plane strain (IPS) shape change for *all* transformations. However, when the IPS shape change has a large shear component, it implies at the very least coherency at the parent/product interface, which must be glissile [14]. A glissile interface is one which can glide without the diffusion of substitutional solutes. From their observations of surface relief, Srinivasan and Wayman in 1969 concluded that coherency is maintained at the interface during the growth of bainite [15].

This means that the iron and substitutional solutes do not diffuse during transformation; this has been verified experimentally on the finest conceivable scale [16]. The effect of substitutional solutes on the transformation behaviour is therefore to alter the thermodynamics of transformation, but the fact that they do not diffuse means that their influence on transformation kinetics is reduced. This has consequences on the time–temperature–transformation (TTT) diagram.

The TTT diagram can be divided into two C–curves, the higher temperature one describing the evolution of reconstructive and the lower the displacive transformations (Fig. 3). The addition of an austenite stabilizing substitutional solute has a much larger effect on the former, because it may retard transformation both by reducing the thermodynamic driving force and by partitioning between the phases by diffusion which can become rate–limiting.

Consequences: Relation to γ Grain Structure

The coordinated movement of atoms associated with displacive transformations cannot in general be sustained across austenite grain boundaries, whereas this is not a restriction when the mechanism is reconstructive. The inability of bainite to cross the austenite grain boundaries therefore leads to a further refinement of the microstructure. This applies to any product generated by a deformation of the austenite. However, there are certain special considerations relevant to bainite, which lead to an exceptional degree of refinement.

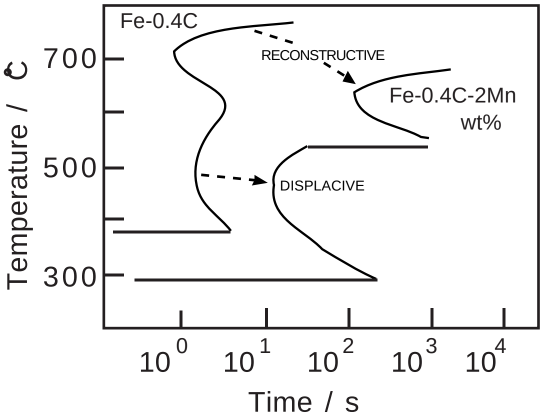

Fig. 3: TTT curves showing that the addition of Mn greatly retards reconstructive transformation relative to the corresponding retardation of the displacive transformation.

Plates of bainite in fact stop growing even before they encounter an austenite grain surface; these tiny plates are called "sub–units" because they grow in clusters called sheaves. Within each sheaf the sub–units are parallel and of identical crystallographic orientation and habit plane. The sub–units are usually separated from each other either by the presence of carbides or residual austenite [2].

The reason why the sub–unit of bainite grows to a restricted size is that the transformation occurs in a temperature regime where the shape deformation cannot be elastically accommodated by the austenite, whose yield strength decreases with increasing temperature. Fig. 4 illustrates the plastic deformation caused in the adjacent austenite by the transformation. The resulting forest of dislocations blocks the progress of the glissile transformation interface, explaining why the sub–unit stops growing even though it may not have encountered an austenite grain surface. In fact, the size can be decreased further by causing the bainite to grow in plastically deformed austenite [17]; there is evidence to suggest that thermomechanically processed bainitic steels benefit from this *mechanical sta-*

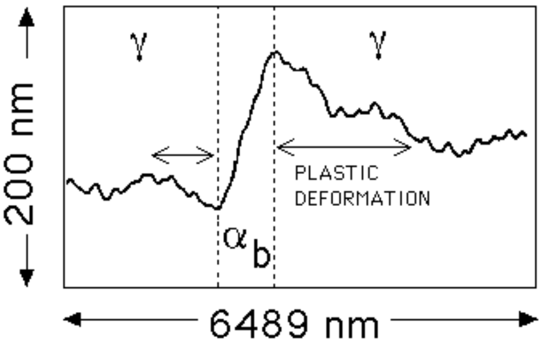

Fig. 4: High–resolution atomic–force microscope plot of the displacements caused by the formation of a single sub–unit of bainite. The surface was flat before transformation. Note the plastic deformation caused in the adjacent austenite [9]

bilisation.

The plastic relaxation of the shape change causes an increase in the dislocation density of bainitic ferrite since any motion of the interface into the deformed austenite will cause the defect structure to be inherited. Thus, although there is considerable scatter in published data, the dislocation density generally decreases as the transformation temperature is increased (Fig. 5). An assessed empirical relationship over the temperature range 570–920 K [18]:

$$\log_{10}\{\rho_D\} = 9.2840 + \frac{6880.73}{T} - \frac{1780360}{T^2} \qquad (1)$$

where T is the transformation temperature in Kelvin, and ρ_D is stated in units of m^{-2}.

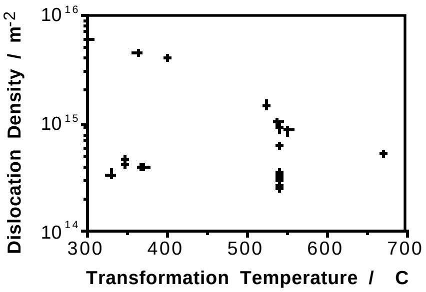

Fig. 5: Dislocation density of martensite, bainite and Widmanstätten ferrite as a function of the transformation temperature [19]

Consequences: Carbon, Incomplete Reaction

It is simple to establish that martensitic transformation is diffusionless, by measuring the local compositions before and after transformation. Bainite forms at somewhat higher temperatures where the carbon can escape out of the plate within a fraction of a second. Its original composition cannot therefore be measured directly.

There are three possibilities. The carbon may partition during growth so that the ferrite may never contain any excess carbon [20]. The growth may on the other hand be diffusionless [21] with carbon being trapped by the advancing interface. Finally, there is an intermediate case in which some carbon may diffuse with the remainder being trapped to leave the ferrite partially supersaturated [22]. It is therefore much more difficult to determine the precise role of carbon during the growth of bainitic ferrite than in martensite.

Diffusionless growth requires that transformation occurs at a temperature below T_0, when the free energy of bainite becomes less than that of austenite of the same composition. A locus of the T_0 temperature as a function of the carbon concentration is called the T_0 curve, an example of which is plotted on the Fe–C phase diagram in Fig. 6. Growth without diffusion can only occur if the carbon concentration of the austenite lies to the left of the T_0 curve.

Suppose that the plate of bainite forms without diffusion, but that any excess carbon is soon afterwards rejected into the residual austenite. The next plate of bainite then has to grow from carbon–enriched austenite (Fig. 7a). This process must cease when the austenite carbon concentration reaches the T_0 curve. The reaction is said to be incomplete, since the austenite has not achieved its equilibrium composition (given by the Ae3 curve) at the point the reaction stops. If on the other hand, the ferrite grows with an equilibrium carbon concentration then the transformation should cease when the austenite carbon concentration reaches the Ae3 curve.

It is found experimentally that the transformation to bainite does indeed stop at the T_0 boundary (Fig. 7b). The balance of the evidence is that the growth of bainite below the $\mathrm{B_S}$ temperature involves the successive nucleation and martensitic growth of sub–units, followed in upper bainite by the diffusion of carbon into the surrounding austenite. The possibility that a small fraction of the carbon is nevertheless partitioned during growth cannot entirely be ruled out, but there is little doubt that the bainite is at first substantially supersaturated with carbon.

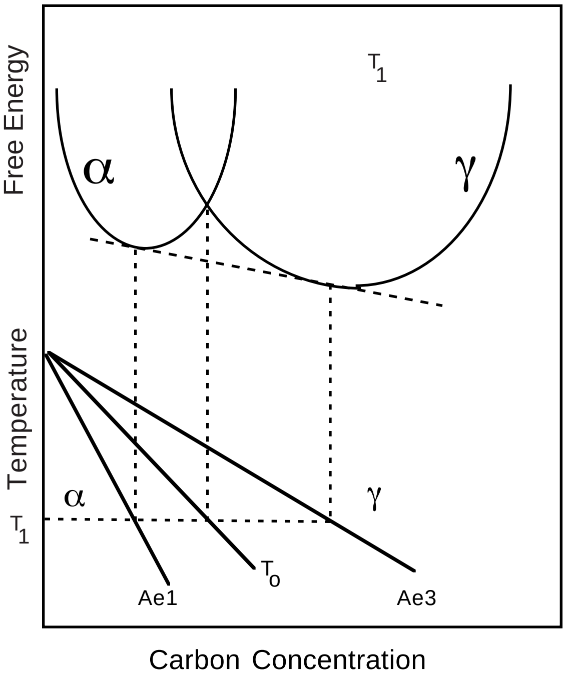

Fig. 6: Schematic illustration of the origin of the T_0 construction on the Fe–C phase diagram. Austenite with a carbon concentration to the left of the T_0 boundary can in principle transform without any diffusion. Diffusionless transformation is thermodynamically impossible if the carbon concentration of the austenite exceeds the T_0 curve.

These conclusions are not significantly modified when the strain energy of transformation is included in the analysis. Strain reduces T_0 to T_0'.

There are two important features of bainite which can be shown by a variety of techniques, *e.g.* dilatometry, electrical resistivity, magnetic measurements and by metallography. Firstly, there is a well defined temperature B_S above which no bainite will form, which has been confirmed for a wide range of alloy steels. The amount of bainite that forms increases as the transformation temperature is reduced below the B_S temperature. The fraction increases during isothermal transformation as a sigmoidal function of time, reaching an asymptotic limit which does not change on prolonged heat treatment even when substantial quantities of austenite remain untransformed. Transformation in fact ceases before the austenite achieves its equilibrium composition, so that the effect is dubbed the "incomplete–reaction phenomenon".

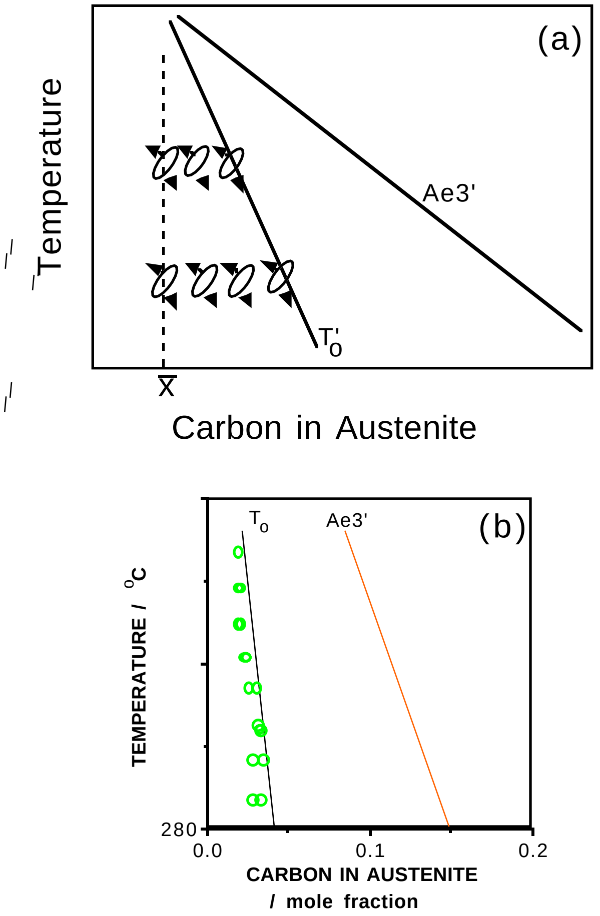

Fig. 7: (a) Illustration of the incomplete reaction phenomenon. During isothermal transformation, a plate of bainite grows without diffusion, then partitions its excess carbon into the residual austenite. The next plate therefore has to grow from carbon–enriched austenite. This process continues until diffusionless transformation becomes impossible when the austenite composition eventually reaches the T_0 boundary. (b) Experimental data showing that the growth of bainite stops when the austenite carbon concentration reaches the T_0 curve (Fe–0.43C–3Mn–2.12Si wt% alloy).

These observations are understood when it is realised that growth must cease if the carbon concentration in the austenite reaches the T_0 curve of the phase diagram. Since this condition is met at ever increasing carbon concentrations when the transformation temperature is reduced, more bainite can form with greater undercoolings below B_S. But the T_0 restriction means that equilibrium, when the austenite has a composition given by the Ae3 phase boundary, can never be reached, as observed experimentally. A bainite–finish temperature B_F is sometimes defined, but this clearly cannot have any fundamental significance.

Alloy Design

High–strength bainitic steels have not in practice been as successful as quenched and tempered martensitic steels, because the coarse cementite particles in bainite are detrimental for toughness. However, it is now known that the precipitation of cementite during bainitic transformation can be suppressed. This is done by alloying the steel with about 1.5 wt% of silicon, which has a very low solubility in cementite and greatly retards its growth.

An interesting microstructure results when this silicon–alloyed steel is transformed into upper bainite. The carbon that is rejected into the residual austenite, instead of precipitating as cementite, remains in the austenite and stabilises it down to ambient temperature. The resulting microstructure consists of fine plates of bainitic ferrite separated by carbon–enriched regions of austenite (Fig. 8).

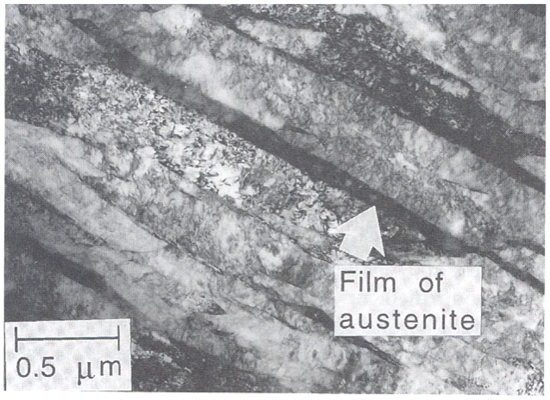

Fig. 8: Transmission electron micrograph of a mixture of bainitic ferrite and stable austenite.

The potential advantages of the mixed microstructure of bainitic ferrite and austenite can be listed as follows:

(a) Cementite is responsible for initiating fracture in high–strength steels. Its absence is expected to make the microstructure more resistant to cleavage failure and void formation.

(b) The bainitic ferrite is almost free of carbon, which is known to embrittle ferritic microstructures.

(c) The microstructure derives its strength from the ultrafine grain size of the ferrite plates, which are less than 1 μm in thickness. It is the thickness of these plates which determines the mean free slip distance, so that the effective grain size is less than a micrometre. This cannot be achieved by any other commercially viable process. Grain refinement is the only method available for simultaneously improving the strength and toughness of steels.

(d) The ductile films of austenite which are intimately dispersed between the plates of ferrite have a crack blunting effect. They further add to toughness by increasing the work of fracture as the austenite is induced to transform to martensite under the influence of the stress field of a propagating crack. This is the TRIP, or transformation–induced plasticity effect.

(e) The diffusion of hydrogen in austenite is slower than in ferrite. The presence of austenite can therefore improve the stress corrosion resistance of the microstructure.

(f) Steels with the bainitic ferrite and austenite microstructure can be obtained without the use of any expensive alloying additions. All that is required is that the silicon concentration should be large enough to suppress cementite.

In spite of these appealing features, the bainitic ferrite/austenite microstructure does not always give the expected good combination of strength and toughness. This is because the relatively large “blocky” regions of austenite between the sheaves of bainite (Fig. 9) readily transform into high–carbon martensite under the influence of stress. This untempered, hard martensite embrittles the steel.

The blocks of austenite are clearly detrimental to toughness, and anything that can be done to reduce their fraction, or increase their stability to martensitic transformation, would be beneficial. Both of these effects are controlled by the T_0' curve of the phase diagram. This curve determines the composition of the austenite at the point where the reaction to bainite stops. By displacing the curve to larger carbon concentrations, both the fraction of bainite that can form, and the carbon concentration of the residual austenite can be increased. Modifications to the T_0' curve can be achieved by altering the alloy composition. It is therefore necessary to calculate the effect of substitutional

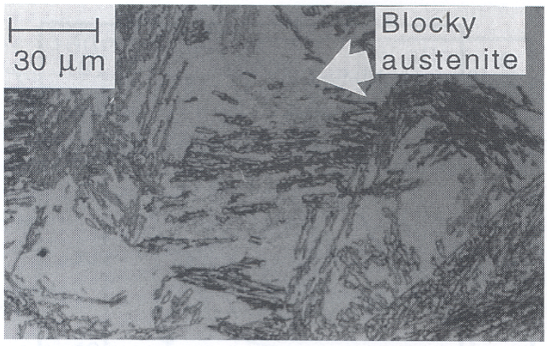

Fig. 9: Optical micrograph of upper bainite in an Fe–0.43C–3Mn–2.02Si wt% showing the blocks of retained austenite between sheaves of bainite.

solutes on the T_0' curve.

Improvement in Toughness

An apparently ideal microstructure consisting of bainitic ferrite and ductile austenite in a Fe–3Mn–2.02Si–0.43C wt% exhibits poor toughness because of the presence of blocky, unstable austenite. It is necessary to increase the amount of bainitic ferrite in the microstructure and to increase the stability of the austenite. Both of these aims can be achieved by changing the substitutional solute concentration such that the T_0' curve is shifted to higher carbon concentrations (*i.e.* T_0' is raised at any given carbon concentration).

It can be demonstrated that the change in the T_0 temperature caused by the addition of a substitutional element (concentration x in atomic %) is given approximately by [23]:

$$\Delta T_0 = \frac{x(b_{NM}\Delta T_{NM} + b_M \Delta T_M)}{b_{NM} + b_M} \tag{2}$$

where the values of b are in Table 3a. The effect of several alloying elements can be approximated by assuming additivity:

$$\Delta T_0 = \frac{\sum_i x_i(b_{NM}\Delta T_{NM_i} + b_M \Delta T_{M_i})}{b_{NM} + b_M} \tag{3}$$

Using these equations and the data from Table 3, we see that for the Fe–3Mn–2.02Si–0.43C wt% (2.97Mn, 3.87Si at.%) alloy, $\Delta T_0 = 116$.

The T_0' curve for a plain carbon steel is over the temperature range of interest, given approximately by [23]

$$T_0'(\text{ K}) \simeq 970 - 80x_C$$

so that for the alloy it is modified to,

$$T_0'(\text{ K}) \simeq 970 - 80x_C - 116 \tag{4}$$

Hence, manganese is seen to have a large effect in depressing the T_0' temperature. An examination of Table 3b shows that one possibility is to replace all of the manganese with nickel. Thus, for a Fe–4Ni–2Si–0.4C wt% (3.69Ni, 3.85Si at.%) alloy, a similar calculation shows that $\Delta T_0 \simeq 72$ so that

$$T_0'(\text{ K}) \simeq 970 - 80x_C - 72 \tag{5}$$

Function	a	b	Temperature Range
$\Delta G_{NM}^{\gamma\alpha}$	–6660	7	$900 > T > 300$ K
$\Delta G_{M}^{\gamma\alpha}$	650	–1	$900 > T > 620$ K
$\Delta G_{M}^{\gamma\alpha}$	0	0	$T < 620$ K

Table 3a: Approximate representations of the magnetic (ΔG_M) and non–magnetic (ΔG_{NM}) free energy components for the $\gamma \rightarrow \alpha$ transformation in pure iron. Each free energy term is given by $\Delta G = a + bT$ J mol^{-1}

Solute	ΔT_M / K per at.%	ΔT_{NM} / K per at.%
Si	–3	0
Mn	–37.5	–39.5
Ni	–6	–18
Mo	–26	–17
Cr	–19	–18
V	–44	–32
Co	19.5	16
Al	8	15
Cu	4.5	–11.5

Table 3b: Values of ΔT_M and ΔT_{NM} for a variety of substitutional solutes (H.I. Aaronson, H. A. Domian and G. M. Pound, *Trans. Met. Soc. AIME,* vol. 236, 1966, 753–780.)

The remarkable improvement in toughness achieved by doing this, without any sacrifice of strength, is illustrated in Fig. 10, along with the T_0' curves as calculated above.

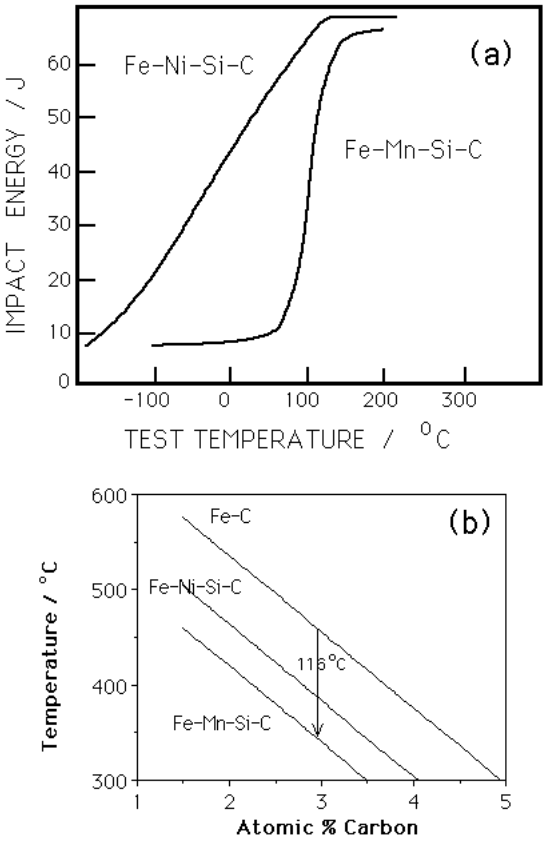

Fig. 10: (a) Experimentally determined impact transition curves showing how the toughness improves as the amount of blocky austenite is reduced. (b) Calculated T_0' curves for the Fe–C, Fe–Mn–Si–C and Fe–Ni–Si–C steels.

Commercial Applications

We demonstrated above, that a knowledge of the mechanism of bainite formation can be exploited towards the design of new steels. These concepts have now been used both in the development of novel ultra–high strength steels with strength and toughness combinations which match or exceed more expensive alloys Fig. 11 [24,25].

A recent major application has been in the development of rail steels which are tough and at the same time extremely resistant to wear [26]. Producers of steel for railway tracks have had the long standing difficulty that the harder they make the rail, the longer it lasts but can increase the wear can be suffered by the rolling stock wheels. The microstructure of conventional rails is based on a mixture of cementite and ferrite in the form of pearlite. The cementite is hard and therefore provides wear resistance, but is at the same time brittle. The new bainitic rail steel is completely free of carbides; it has a much higher toughness while at the same time being harder due to the fine grain size and the presence of some martensite and retained austenite. Tests show that it has remarkable wear resistance, reduces wear on the wheels, is tough and weldable (Fig. 12). The steel is now commercialised.

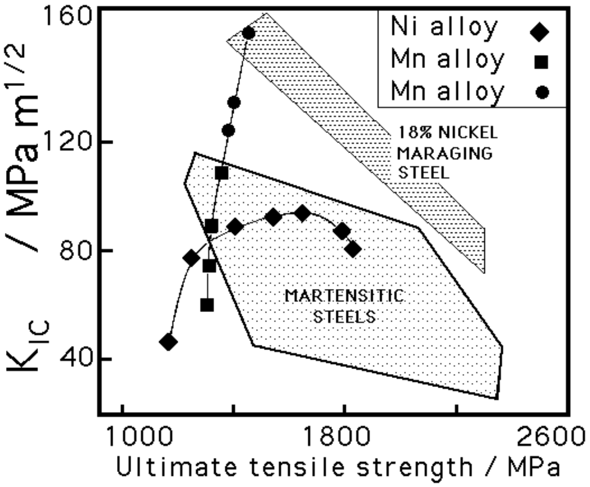

Fig. 11: Some of the new cheap bainitic steels match the properties of expensive alloys [25]

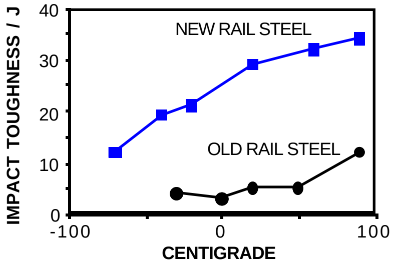

Fig. 12: Toughness of new rail steel against conventional rail steels

Bainite: Shape Memory Effect

Professor Wayman has contributed greatly towards the modern understanding of shape memory effects. It is instructive to examine whether bainite can exhibit a similar phenomenon – we begin with a description given by Wayman of the SME [6].

An object in the low temperature martensitic condition when deformed and then unstressed regains its original shape on heating from the austenite–start to austenite–finish temperature, as the parent phase grows at the expense of the martensite. The strains (6–8%) describing the original deformation of the martensite are completely recovered. The recovery of the original shape is entirely due to the reverse transformation of the deformed martensite.

The SME effect can be explained as follows [27], Saburi *et al.* [28]. A single crystal of the parent phase will usually transform into 24 orientations of martensite. But when this mixture of many variants of martensite is deformed, the microstructure changes into a single orientation of martensite. This change occurs by the motion of martensite–martensite boundaries, during which the variant most compatible with the applied deformation grows at the expense of the others. The motion of the martensite–martensite boundaries involves some twinning/detwinning deformation, because the twins are in fact simply other orientations of martensite. It has been shown that twinning can convert one orientation (variant) of martensite to another [28]. Consequently, when a fully martensitic mixture of many orientations of martensite is deformed, one plate orientation grows at the expense of the others. The favoured martensite is that whose shear component of the shape deformation permits the maximum elongation of the specimen in the direction of the tensile axis [29].

Although a single crystal of the parent phase transforms into many orientations of martensite, the reverse situation does not occur. The highly symmetric parent phase (usually cubic) has many crystallographically equivalent ways in which the Bain distortion can occur, giving numerous variants of martensite. On the other hand, the relatively unsymmetric martensite does not have such a multiplicity of choices, so that only the original orientation of parent phase can form. The single crystal of martensite "unshears" to form a single crystal of the original parent phase, thereby restoring the specimen to its original shape.

The entire deformation is therefore due solely to martensite variant reorientation. Any other form of deformation (*e.g.* ordinary slip) tends to destroy the shape memory effect. We have emphasised that the microstructure of bainite is as it is because the shape change is plastically accommodated. This alone is sufficient to rule out any shape memory behaviour because the interface is rendered sessile by the resulting dislocation debris. A further hindrance is the fact that the carbon is partitioned into the austenite, making a strict reversal of the transformation impossible.

Conclusions

There is more that we now understand about bainite, then what is confusing. This has permitted the construction of satisfying theory, which can be exploited in the design of alloys for industry, and in making predictions verifiable by experiment. Some of the applications of bainite are presented in Fig. 13.

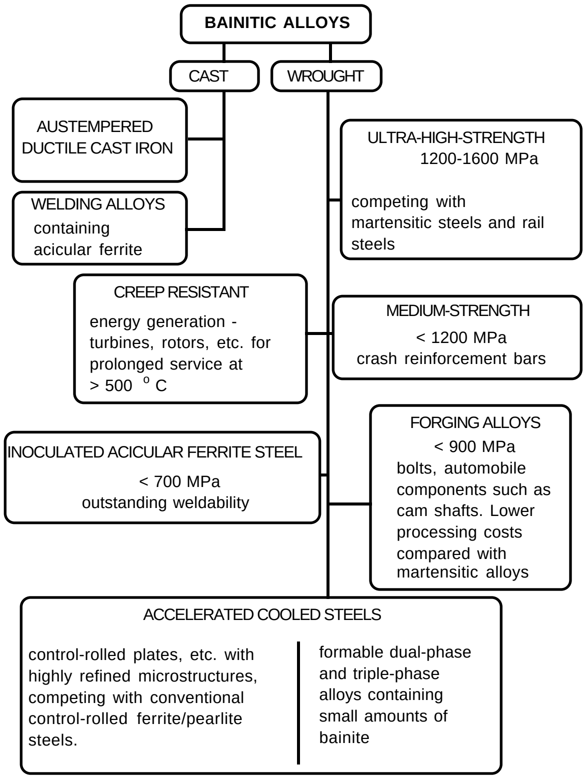

Fig. 13: Modern bainitic steels [2]

This happy state of affairs seems unlikely without the elegant work done over many decades by Professor Wayman and his many disciples. This work vividly influenced what is understood by the term "bainite". It is indeed a pleasure to contribute to this commemorative meeting for an intellectual giant who has always shown the way forward.

Acknowledgments

I am grateful to the Royal Society for a Leverhulme Trust Senior Research Fellowship, to the EPSRC for a research grant on transformation plasticity, and to Professor Alan Windle for the provision of laboratory facilities at the University of Cambridge.

References

1. C. M. Wayman, "The growth of martensite since E C Bain (1924) - some milestones", *Materials Forum* 56-58 (1990) 1–32.

2. H. K. D. H. Bhadeshia, “Bainite in Steels”, Institute of Materials, London, (1992) 1–458.

3. H. K. D. H. Bhadeshia, “Diffusional formation of ferrite in iron and its alloys”, *Progress in Materials Science* 29 (1985) 321–386.

4. E. C. Bain, “The Nature of Martensite”, *Trans. AIME* 70 (1924) 25–46.

5. C. M. Wayman and A. J. Altstetter, “Martensite in zone–refined iron”, *Acta Metallurgica* 10 (1962) 992–993.

6. C. M. Wayman and H. K. D. H. Bhadeshia, “Non–Diffusive Transformations”, *Physical Metallurgy*, eds R. W. Cahn and P. Hassen, Elsevier, Holland, (1996) 1507–1554.

7. J. D. Watson and P. G. McDougall, “The crystallography of Widmanstätten ferrite”, *Acta Metallurgica* 21 (1973) 961–973.

8. B. J. P. Sandvik, “The bainite reaction in Fe–Si–C alloys”, *Metallurgical Transactions A* 13A (1982) 777–800.

9. E. Swallow and H. K. D. H. Bhadeshia, “High resolution observations of the displacements caused by bainitic transformation”, *Materials Science and Technology* (1996) in press.

10. C. M. Wayman, *Introduction to the Crystallography of Martensite*, Macmillan, New York, (1964).

11. D. P. Dunne and C. M. Wayman, “The crystallography of ferrous martensite”, *Metallurgical Transactions* 2 (1971) 2327–2341.

12. J. F. Knott, *Fundamentals of Fracture Toughness*, Butterworths, London (1973).

13. J. Daigne, M. Guttman and J. P. Naylor, “Influence of lath boundaries and carbide distribution on yield strength of tempered martensite steels” *Materials Science and Engineering*56 (1982) 1–10.

14. J. W. Christian, “Crystallographic Theories, Interface Structure and Surface Relief”, *Metallurgical Transactions* 25A (1994) 1821–1839.

15. G. R. Srinivasan and C. M. Wayman, “Crystallography of the bainite transformation”, *Acta Metallurgica* 16 (1968) 621–636.

16. H. K. D. H. Bhadeshia and A. R. Waugh, “An atom–probe study of the incomplete reaction phenomenon”, *Acta Metallurgica* 30 (1982) 775–784.

17. P. Shipway and H. K. D. H. Bhadeshia, “Mechanical stabilization of bainite”, *Materials Science and Technology* 11 (1995) 1116–1128.

18. M. Takahashi and H. K. D. H. Bhadeshia, “Model for the transition from upper to lower bainite”, *Materials Science and Technology* 6 (1990) 592–603.

19. H. K. D. H. Bhadeshia, “Possible effects of stress on steel weld microstructures”, *Mathematical Modelling of Weld Phenomena II,* ed H. Cerjak, Insitute of Materials, London, (1995) 71–118.

20. A. Hultgren, “Isothermal transformation of austenite”, *Trans. A.S.M.,* 39 (1947) 915–1005.

21. J. R. Vilella, G. E. Guellich and E. C. Bain, “On naming the aggregate constituents in steel”, *Trans. A.S.M.,* 24 (1936) 225–261.

22. N. A. Smirenskaya, L. I. Kogan and R. I Entin, “Similarities and differences in the reactions of Widmanstätten ferrite and bainite formation”, *Fiz. Metal. Metalloved.,* 41 (1976) 1019–1028.

23. R. W. K. Honeycombe and H. K. D. H. Bhadeshia, *Steels: Microstructure & Properties,* 2nd edition, Edward Arnold, London (1995) 306–313.

24. H. K. D. H. Bhadeshia and D. V. Edmonds, “Bainite in silicon steels: a new composition property approach”, *Metal Science* 17 (1983) 411–425.

25. V. T. T. Miihkinen and D. V. Edmonds, “Fracture toughness of two experimental high strength bainitic steels”, *Materials Science and Technology* 3 (1987) 441.

26. H. K. D. H. Bhadeshia and V. J. Jerath, unpublished research, 1989–1996.

27. T. A. Schroeder and C. M. Wayman, “The formation of martensite & the mechanism of the shape memory effect in single crystal of Cu–Zn”, *Acta Metallurgica*, 25 (1977) 1375–1385.

28. T. Saburi, S. Nenno and C. M. Wayman, “Shape memory mechanisms in alloys”, *Proc. ICOMAT ’79*, Alpine Press, Boston, Massachusetts (1979) 619–632.