Roughness of bainite

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A bainite sheaf does not have a simple geometry, making it difficult to characterise or calculate its fundamental properties, such as the total amount of interfacial area per unit volume. The sheaf is, in the language of fractals, a rough object in which the area is a function of how the measurements are made. Micrographs taken at a variety of resolutions have been analysed to reveal how the area scales with resolution. It is found that although the interface is rough, it is far less so when compared with what might be expected from a fractal object. In other words, the ideal fractal, where self-similarity propagates over an infinite range of observation does not apply to the bainite sheaf.

Keywords: Bainite, Sheaf, Subunits, Roughness, Fractal, Stereology

Introduction

A fascinating aspect of the bainite reaction in steels is that it occurs by a subunit mechanism in which a platelet of ferrite grows to a limited size, even though there is no impingement with obstacles such as austenite grain boundaries. The transformation then propagates by the nucleation and growth of another subunit, the collection of subunits being known as a sheaf.[1] The reason why each platelet has a limited size is that the shape deformation accompanying transformation is plastically accommodated in the austenite beside the plate.[2,3] This results in the creation of an intense dislocation debris which renders the interface immobile, and hence the subunit mechanism of sheaf growth.

The subunits within an individual bainite sheaf are in fact contiguous, as shown by the tracing in Fig. 1, made using an actual transmission electron microscope image of a bainite sheaf.[2,4,5] It follows that the shape of the austenite γ/bainitic ferrite α_b interface is far from smooth; there are many published micrographs showing the convoluted outline of a bainite sheaf (e.g. Figs. 2 and 3 of Ref. 4).

A smooth object is one whose properties do not change with resolution, for example, the perimeter of a perfect circle. Measures such as perimeter and surface area are not well defined for objects which are rough, because they depend on the resolution of the measuring technique. The surface area of a brick is a function of the method used to measure the area; the brick is said to be a rough object.

We examine here the roughness of the surface of a bainite sheaf, using elementary fractal analysis, as reviewed recently in Ref. 6.

Method

The essence of the problem is to measure on micrographs, the perimeter presented by bainite sheaves per unit of micrograph area, using a variety of resolutions. The authors have accumulated many micrographs of bainite at a variety of resolutions over the years, especially for a Fe–0·43C–2·02Si–3Mn (wt-%) steel, austenitised at 1200°C for 10 min followed by partial isothermal transformation to bainite over the temperature range of 192–295°C. These were exploited in the study using two methods.

Box counting

The fractal dimension D of a two-dimensional pattern can be determined by superimposing a square grid on the image,[6] and counting the number of boxes per unit area N_i that includes the interface, given a particular square size (i.e. resolution) ε_i

$$D=\frac{\ln N_i}{\ln \varepsilon_i^{-1}} \tag{1}$$

with a 95% confidence limit given by $2\sigma=\pm 2/N_i^{1/2}$. D can therefore be obtained from the gradient of a plot of $\ln \varepsilon_i^{-1}$ against $\ln N_i$. Figure 2 thus gives $D=1{\cdot}71$. Because the analysis is based on micrographs, the D value is for a one-dimensional fractal curve, $D_{1D}\approx 1{\cdot}71$. A bainite sheaf is in practice a three-dimensional object, therefore it is estimated that $D_{3D}\simeq 2+D_{1D}=3{\cdot}71$.

Intercept method

A mean lineal intercept is another parameter which can be used to characterise the α_b/γ sheaf interface. Random lines are superimposed on the micrographs and the segments resulting from the intersection of the lines with the sheaf interface that is measured and averaged by the total number of intercepts. $\bar{L}$ is simply related to the amount of surface per unit volume S_V by the stereological relation[7]

$$S_V=\frac{2}{\bar{L}} \tag{2}$$

S_V is related to the measurement resolution

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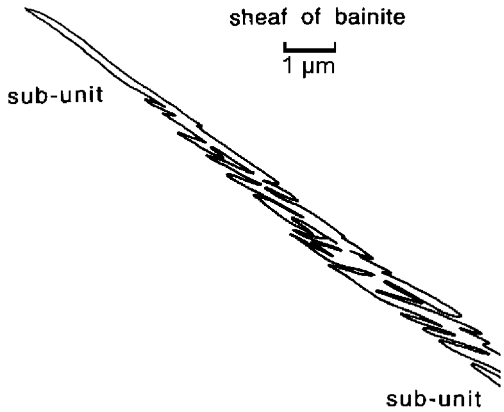
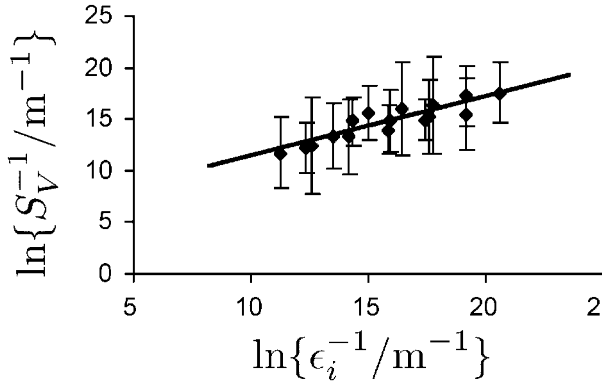


1 **Outline of subunits near tip of bainite sheaf**



3 **Plot to determine fractal dimension of traces of α_b/γ sheaf interfaces determined using intercept method**

$$S_V = S_0 \varepsilon^{D_T - D} \tag{3}$$

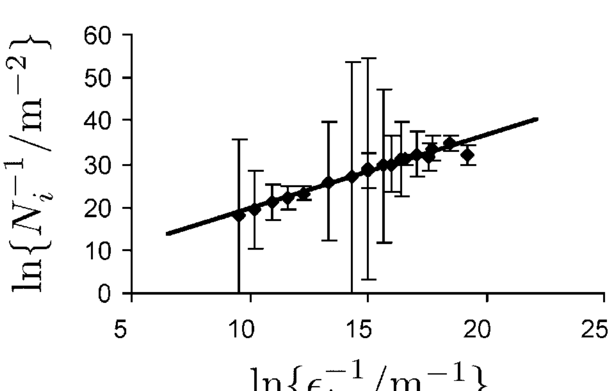
where S_0 is the surface/volume ratio of a smooth object of topological dimension D_T and D is the corresponding fractal dimension of a rough object. For a smooth object, $D_T=D$. The 95% confidence limits in the lineal intercept measurements (and hence in S_V), can be estimated as $2\sigma = \pm 2\sigma_L/\bar{L}N_i^{1/2}$, where σ_L is the standard deviation of the intercepts.

A plot of $\ln S_V$ versus $\ln(\varepsilon^{-1})$ (Fig. 3) gave $D-D_T=0{\cdot}59$ so that, because $D_T=3$ for a three-dimensional sheaf. It is interesting that D is similar to the value of 3·71 found using the box counting method. Equation (3) along with the equation of the line in Fig. 3 can now be used to determine the surface area due to sheaves of bainite as a function of the resolution.

Model for fractal sheaf

Suppose that a bainite sheaf has a fractal character, i.e. smaller plates are observed within larger plates, and that this continues to be the case at every step of magnification. It would be useful to determine the fractal dimension in such a scenario for comparison against the experimental data presented above.

Consider each platelet (subunit) within a sheaf to have a thickness t and an elliptical shape with semiaxes a and b. A sheaf is formed by stacking planar arrays of these ellipses with intervening regions of retained austenite (Fig. 4). The thickness of the austenite layers is defined by the fraction of bainitic ferrite in the microstructure.



2 **Plot to determine fractal dimension of traces of α_b/γ sheaf interfaces determined using box counting method**

In a fractal scenario, each subunit would contain smaller platelets and this process would continue *ad infinitum* as the magnification is increased. Each set of identical subunits is henceforth called a generation.

Construction of the fractal requires scale invariance under isotropic dilatation (a, b and t simply scale to λa, λb and λt where λ is a constant).

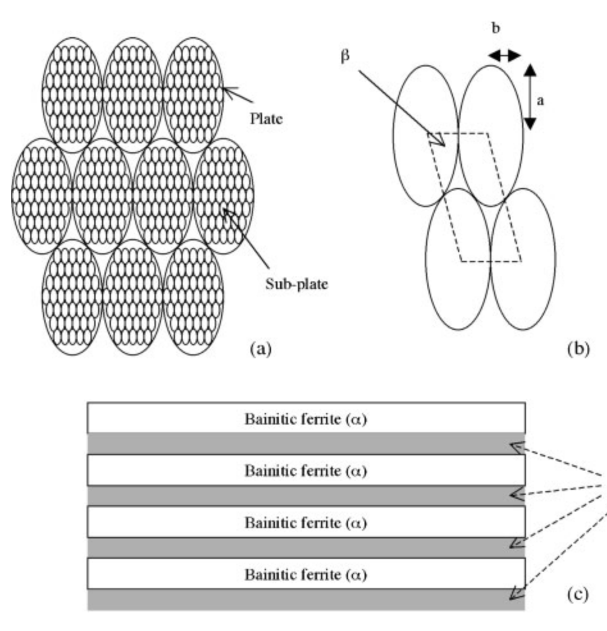
The problem now is to find how the surface area of the α_b/γ interfacial area within the sheaf scales with resolution. After some lengthy algebra,[8] for the nth generation

$$N_{V_n} = N_{V_1}(\lambda^3)^{n-1} \tag{4}$$

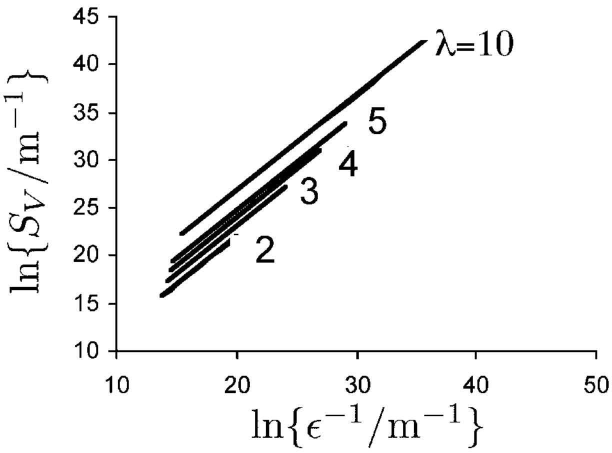
where N_{V_n} is the number of nth generation platelets per unit volume

$$V_n = \pi a_n b_n t_n N_{V_1}(\lambda^3)^{n-1} \tag{5}$$

where V_n is the total volume of the nth generation of platelets and the amount of α_b/γ interfacial area per unit volume, owing to the nth generation of platelets is given



4 **Hypothetical sheaf consisting of planar array of elliptical plates (subunits), with planar arrays stacked with intervening retained austenite in three dimensions: each elliptical platelet will contain other generations of smaller platelets to form fractal pattern**



5 Surface per unit volume as function of resolution in this case with $\varepsilon=a_n$ and parameter λ

by

$$S_{V_n}=\frac{\left\{2\pi a_n b_n+2t_{\alpha_n}\pi\left[\frac{1}{2}\left(a_n^2+b_n^2\right)\right]^{1/2}\right\}N_{V_1}(\lambda^3)^{n-1}}{L_\gamma^2 T_\gamma} \tag{6}$$

where L_γ is the length of a square sectioned austenite grain, T_γ is the thickness of the austenite grain and t_α is the thickness of a planar array of ellipses, so that $t-t_\alpha=t_\gamma$ becomes the thickness of the intervening austenite layer. The results from equation (6) using $L_\gamma=100$ µm, $T_\gamma=0{\cdot}2$ µm, $t_\gamma/t_\alpha=0{\cdot}2$ and the parameters listed in Table 1, are given in Fig. 5. Because the slopes of all the lines is unity, it follows using equation (3) that $D=4$. An identical fractal dimension was found using an array of square instead of elliptical plates,[8] therefore the details are not reproduced here.

Discussion

The important result is that the surface area of a bainite sheaf as measured experimentally does not increase as rapidly as would be expected when the fractal dimension is 4. This is physically correct because there is no mechanistic reason why the subunit structure should extend to ever finer scales.

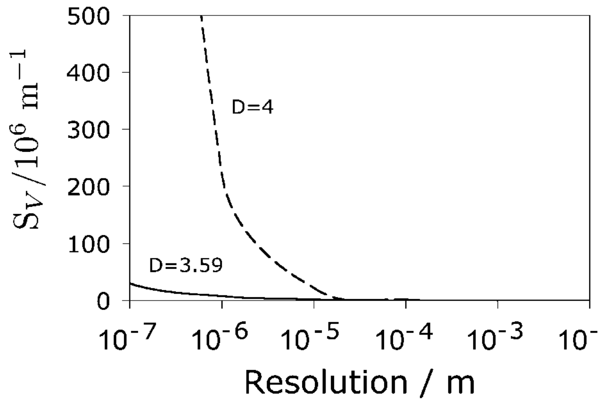
The practical difference between a fractal dimension of 4 as opposed to 3·59 is illustrated in Fig. 6. In this, the equation of the straight line in Fig. 3, i.e.

$$\ln S_V=0{\cdot}59\ln\varepsilon^{-1}+5{\cdot}4 \tag{7}$$

was used to calculate how the amount of α_b/γ interface would scale with resolution, and the case for $D=4$ was calculated using a slope of unity instead of 0·59 in equation (7). It is evident that S_V rises far more rapidly for $D=4$ as resolution is increased, and a point will be reached where all the free energy available is consumed simply in the creation of interfaces. Therefore, although

Table 1 Data used to generate Fig. 5

λ	*a*, µm	*b*, µm	t_α, µm
2	50	1	0·1
3	33·3	0·66	0·066
4	25	0·5	0·05
5	20	0·4	0·04
10	5	0·1	0·01



6 Comparison of measured variation in S_V versus that calculated for an ideal fractal

the bainite sheaf has a rough interface, it does not possess a fractal character. This means that there must be a limited number of generations of subunits. From a physical point of view, it is expected and observed that there should exist only two generations, leading to a bimodal distribution of subunit sizes.[9] The largest generation represents the platelets which have formed to the point where their growth is stifled by mechanical stabilisation.[2,10] The much smaller platelets are the suboperational embryos which have yet to make it into the rapid growth stage.[9]

Summary

The fractal dimension of the surfaces of bainite sheaves has been estimated to be approximately 3·6–3·7, determined by making measurements over a large range of spatial resolutions ($10^{-5}-10^{-9}$ m). Modelling a bainite sheaf as a fractal in which self-similarity propagates over an infinite range of observations gives a fractal dimension of 4. Such a character is not physically reasonable. Comparison between the measurements and model indicates that the real sheaf contains much less detail and roughness than the perfect fractal. The bainite sheaf cannot therefore be regarded as consisting of many generations of subunits, but at the same time cannot be seen as a smooth object with a fixed surface area independent of magnification.

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