Examples Class 1: Stereographic Projections

The Stereographic Projection

Directions or plane normals drawn from a centre point can be projected outwards to the surface of a circumscribing sphere, to give a set of dots on the sphere, Fig. 1. These dots can then be projected through either the north or south pole (depending on whether the points lie in the southern or northern hemispheres respectively) onto a sheet of paper to represent angular relationships.

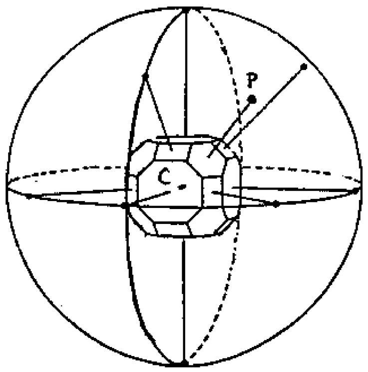

Figure 1: Directions emanating from the centre 'C' intersect the sphere at points such as 'P'.

The advantage of such a projection is that both great and small circles project as true circular arcs, and that the angle between two lines on the surface of the sphere is equal to that between the projections of those lines. The circumference of the projection forms the *primitive circle*, on which all poles that lie horizontal project (Fig. 2).

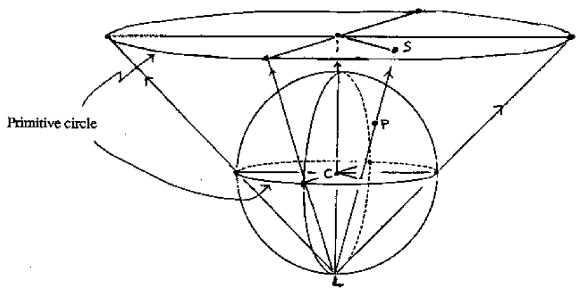

Figure 2: The primitive circle on the equitorial plane.

You will use the apparatus provided (Fig. 3) to investigate these properties – **be careful not to let the plastic hemisphere touch the bulb or it may melt.** Lines or dots drawn on the plastic hemisphere are projected as shadows cast by the light from the 'point' light source, onto the hinged perspex screen. The shadows can be recorded onto tracing paper clipped to the screen using the metal frame whcih lifts away from the perspex.

The light source can to a limited extent be moved up and down. The hemisphere can be rotated about an axis normal to that of the projection, through an angle which can be measured approximately, and within angular limits determined by contact with the bulb assembly. The hemisphere can be detached and replaced.

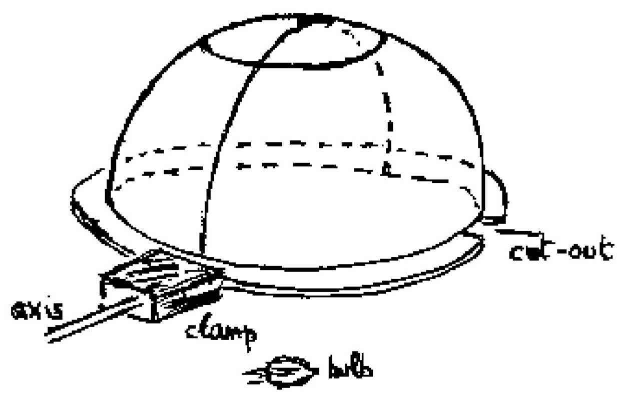

Figure 3: Hemisphere, bulb and clamp in the shadow projector.

1. Unmount a hemisphere and making use of the bendy plastic strip, draw a great circle through the topmost point on the hemisphere, in a plane perpendicular to the cut–out marked on Fig. 3. Use only the washable-ink pens provided. Mark the topmost with a dot. Draw a circle of radius $\simeq 25$ mm, centred on the topmost point, using the perspex sheet with a circle cut out of it; what angle does this cover?

2. Attach the hemisphere bulging upwards, as in Fig. 3. Position the bulb up/down so that it would lie on the surface of the lower hemisphere (not present). This can be judged by turning the hemisphere through 90°: the bulb should fit in the cut–out of the rim.

3. Clip a piece of tracing paper to the perspex sheet. On the paper mark the projection of the centre dot and draw around it the primitive circle at a radius of 105 mm. Next, draw the shadows cast by the great circle and the small circle traced on the plastic hemisphere as the hemisphere is turned at intervals of 30° from the horizontal towards the vertical position. Points below the horizontal project outside the primitive. Notice that circles project as circles, even beyond the primitive, but that the geometrical centres are not at the angular centres.

   Angular truth is maintained. Thus, the angle of 90° between the lines on the hemisphere remains the same in projection.

4. Unmount the sphere and add some more lines: thinking of the cut–out point as the Earth's North Pole, add lines of latitude and longitude at 30° intervals. On a blank tracing paper, sketch the projection obtained with the hemisphere horizontal. Such a net can be used to plot angular positions on a projection and is part of the Wulff net. Confirm that circles remain circular and that angular truth is maintained.

5. On a clean hemisphere, imagining a cube to be placed at the centre of the hemisphere as in Fig. 1, *i.e.* with the direction [001] defining the topmost point on the hemisphere (wooden cubes are provided to help you with this), mark on the following:
   - the points on the hemisphere where the normals to the cube faces will intersect the hemisphere;
   - using the bendy plastic strip, three great circles joining these dots.
     What features of the cube do these represent?
6. Put the hemisphere in a horizontal position on the projector and restore the bulb to the position for stereographic projection. On a clean piece of paper trace the projections of the dots and great circles. The dots are the projections of the {100} normals to the faces of the cube, whereas the great circles are the projections of planes drawn through the centre of the model parallel to the faces. These are also planes of symmetry of the cube.
7. Unmount the hemisphere and place it over a wooden model of a cube. On the cube there are six {110} planes at 45° to those already projected. Draw the corresponding great circles on the hemisphere, with the bendy plastic strip.
8. Return the hemisphere to the projector and to the same piece of paper in the same orientation as before add the projections of these great circles to your stereographic projection.
9. You have now made an outline of a cubic stereogram centred on 001 which can show the symmetry elements and on which the angular relations between various crystallographic directions can be plotted and measured accurately.

The stereographic projection can be made onto any plane perpendicular to the line, the only difference being the magnification. In the shadow projector, the plane has necessarily been chosen above the hemisphere, but in all other applications we choose to project onto the plane through the centre, C, of the sphere.

Sketch Sterograms

The corners of a wooden cube are being cut off to make small equal faces of the form {111}. The first four new faces are cut in the order (a) (111); (b) $(11\bar{1})$; (c) $(\bar{1}11)$ and (d) $(\bar{1}1\bar{1})$.

1. Sketch stereographic projections, all in the same orientation with 001 at the centre, displaying the faces and the symmetry elements of the model before cutting, and after each new face is made.
2. Using the table of crystal systems in the Data Book in which defining symmetries are given for each crystal system, name the crystal system to which the model will belong after each cut