Answers to Examples Class 2

Calcium Chloride

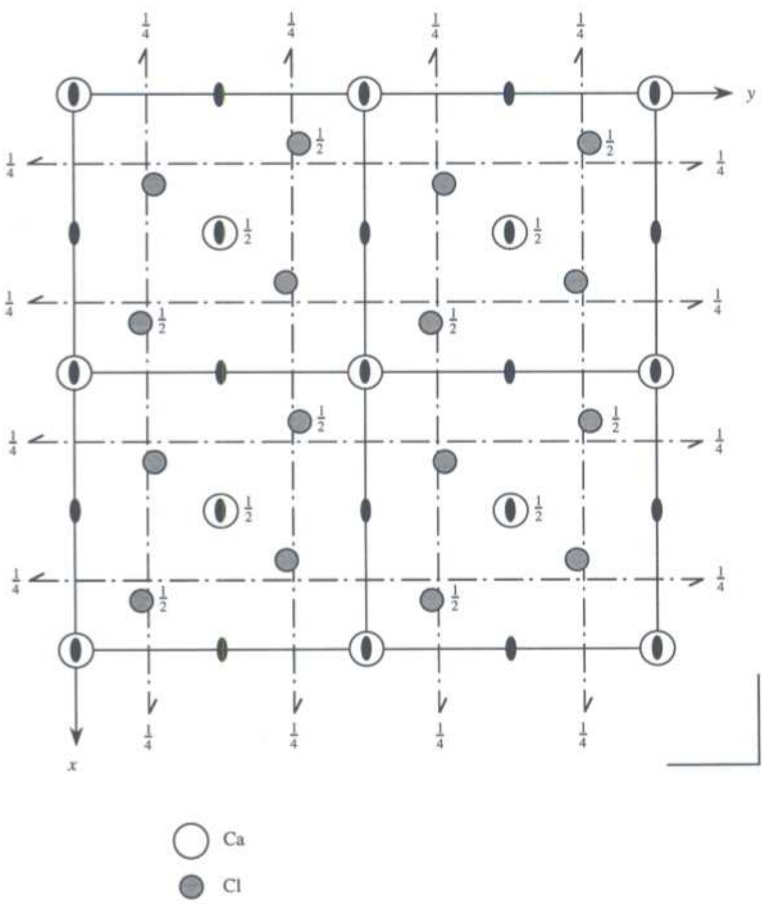

Figure 1: Structure projected along [001]. The Bravais lattice is P (the Ca at the origin has a different environment to that at the centre of the unit cell. Diads parallel to [001] go through each Ca and down the centres of the four sides of the cell containing [001]. (002) planes at $z = 0$ and $\frac{1}{2}$ are mirrors. (200) diagonal glide (n) planes cut at $x = \frac{1}{4}$ and $x = \frac{3}{4}$. (020) diagonal glide (n) planes cut at $y = \frac{1}{4}$ and $y = \frac{3}{4}$. 2_1 screw axes lie in the glide planes at heights $z = \frac{1}{4}$ and $z = \frac{3}{4}$ (only the former is illustrated).

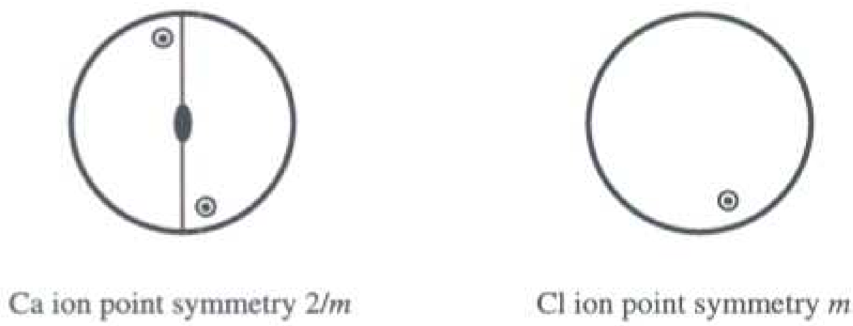

Figure 2: Translational symmetry neglected in determining point group, which is mmm. Point symmetry of Ca is $2/m$ and that of Cl is m.

Martensite

The strain which transforms the structure of austenite into martensite is the Bain strain, which involves a compression along one edge and uniform expansion along the other two edges of the unit cell of austenite.

Suppose that the austenite is represented as a sphere with its unit cell edges denoted by the vectors $\mathbf{a_i}$ with $i = 1, 2, 3$, as illustrated in Fig. 3a,b. The Bain strain changes the sphere into an ellipsoid of revolution about $\mathbf{a_1}$. There are no lines in the $(0\ 0\ 1)_\gamma$ plane which are undistorted. However, it is possible to find lines such as wx and yz are undistorted by the deformation, but are rotated to the new positions $w'x'$ and $y'z'$. Since they are rotated by the Bain deformation they are not invariant–lines. In fact, the Bain strain does not produced an invariant–line strain. It can be converted into an invariant–line strain by adding a rigid body rotation as illustrated in Fig. 3c.

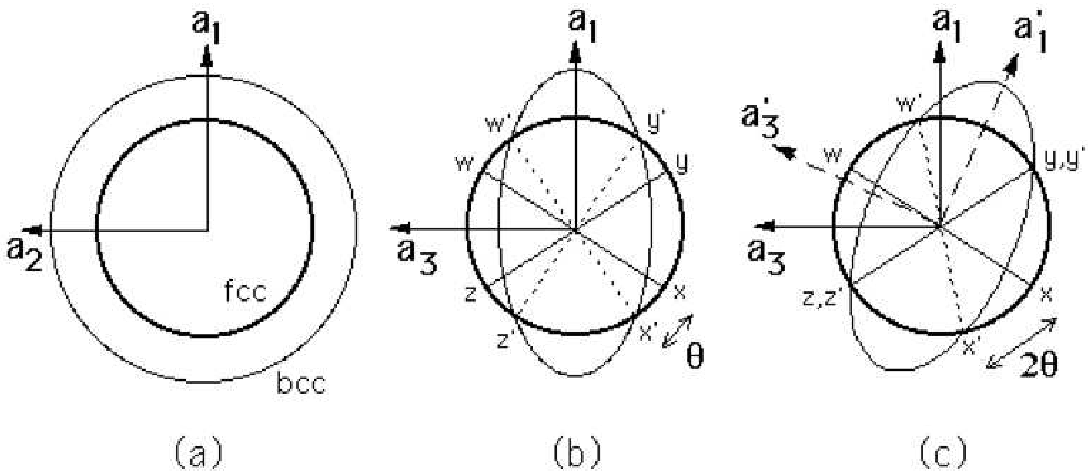

Figure 3: Bain strain and rigid body rotation.

It is also apparent from Fig. 3c that there is no possible rotation which would convert **B** into an invariant–plane strain because there is no rotation capable of making two of the non–parallel undistorted lines into invariant–lines. Thus, it is impossible to convert austenite into α' martensite by a strain which is an invariant–plane strain. A corollary to this statement is that the two crystals cannot ever be joined at an interface which is fully coherent and stress–free.

Orientation Relationships

Inspection of the matrix reveals that

$$J_{11} + J_{22} + J_{33} = 1 + 2\cos\theta \tag{1}$$

and the components of the unit vector $\mathbf{u}$ along the axis of rotation are given by

$$\begin{aligned} u_1 &= (J_{23} - J_{32})/2\sin\theta \\ u_2 &= (J_{31} - J_{13})/2\sin\theta \\ u_3 &= (J_{12} - J_{21})/2\sin\theta \end{aligned} \tag{2}$$

It follows that the matrix describes a rotation of 90° about $[001]_X$, and hence represents a symmetry operation so that $\Sigma = 1$.