Answers to Question Sheet 3, Solid–State Transformations

1. In the absence of work hardening, and assuming that only 5% of the plastic work remains the the metal, the stored energy per unit volume E due to plastic strain ϵ at a stress σ is given by

$$E = \sigma \times \epsilon \times 0.05 = (0.1 \times 10^9) \times 0.4 \times 0.05 = 2 \times 10^6\,\mathrm{J\,m^{-3}}$$

The interfacial energy per unit area, γ, is $0.5\,\mathrm{J\,m^{-2}}$ so for a spherical nucleus the critical radius r^* and the activation energy G^* are given by

$$r^* = 2\gamma/E = 5 \times 10^{-7}\,\mathrm{m} \qquad G^* = \frac{16\pi\gamma^3}{3E^2} = 0.52 \times 10^{-12}\,\mathrm{J}$$

The critical size is within the resolution of a light microscope. This implies that if you were watching the sample you would be able to see recrystallisation nuclei appearing at random! But the activation energy is so large compared with thermal energy $kT \simeq 1.4 \times 10^{-20}$ J at say 1000 K that the probability of such fluctuations is inconceivable.

The conclusion is the recrystallisation does not nucleate by the homogeneous formation of a strain–free grain but by the bowing of existing boundaries as discussed in the lectures.

2. The stored energy is obtained by summing the line energies of all the dislocations present. It can be expressed as a stress σ_R ($\equiv$ energy per unit volume) :

$$\sigma_R \simeq \frac{Gb^2}{2}\Delta\rho = \frac{30 \times 10^9 (3 \times 10^{-10})^2}{2} \times 10^{15} = 1.35\,\mathrm{MPa}$$

where $\Delta\rho$ is the change in dislocation density. The Burgers vector of the dislocations in aluminium is $\frac{a}{2} < 1\ 1\ 0 >= 0.3\,\mathrm{nm}$, where a is the lattice parameter. The Zener pinning stress σ_Z opposing the migration of the boundaries is

$$\sigma_Z = \frac{3}{2}\frac{\gamma f}{r} = \frac{3}{2} \times \frac{0.3 \times 0.012}{50 \times 10^{-9}} = 0.108\,\mathrm{MPa}$$

The particle pinning is not therefore strong enough to restrict recrystallisation.

3. $M24$ is an overaged alloy so the precipitates are visible in an optical microscope (see on–line library). There are precipitate–free zones at the grain boundaries. Such zones are detrimental in two major respects: they reduce the corrosion resistance since the chemical potentials are different in the zones when compared with the matrix; they are weak due to the absence of precipitates.

They can be avoided by a two–stage ageing treatment in which nucleation is stimulated everywhere by ageing at a low temperature (high driving force) and then raising the temperature to allow the precipitates to grow. The last temperature has to be chosen with care to avoid reversion.

4. The extent of the diffusion field increases as ever more solute is partitioned. The partitioned solute therefore has to diffuse over longer distances with increasing precipitate size, thereby reducing the growth rate.

The derivation relies on the fact that the rate at which oxygen is used up in forming oxide must be balanced by the rate at which it arrives at the Cu/O interface:

$$(C^{OM} - C^{MO})\frac{\partial z}{\partial t} = D\frac{C^{OA} - C^{OM}}{z} \qquad \text{so that} \qquad z^2 \propto t$$

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