Rolling contact stresses between two rigid, axial and flat cylinders

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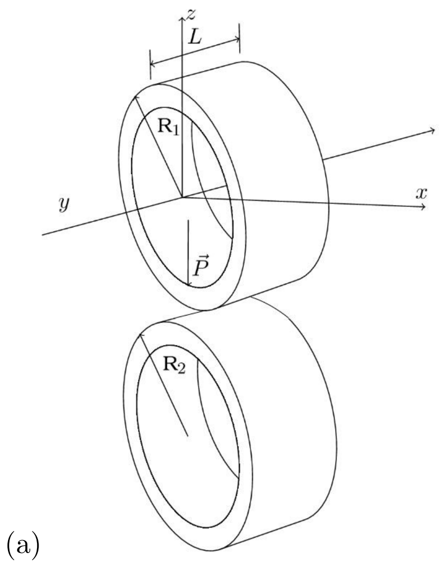
1 Statement of Purpose of the code

The code is written to calculate the distribution of forces and stresses during rolling contact of to rigid, parallel and flat cylinders assuming Hertzian contact between the mating surfaces under various conditions of slip. It aims to estimate the magnitude and distribution of normal and tangential forces acting over the Hertzian contact half-width and subsequently calculates the tangential, normal and shear stresses under plane strain condition. A range of stress distribution from perfect rolling to perfect sliding between mating cylinders can be calculated.

2 Distribution of force over the Hertzian contact width

The schematic of the twin-disc set up is shown in Fig. 1. The discs compressed against each other will



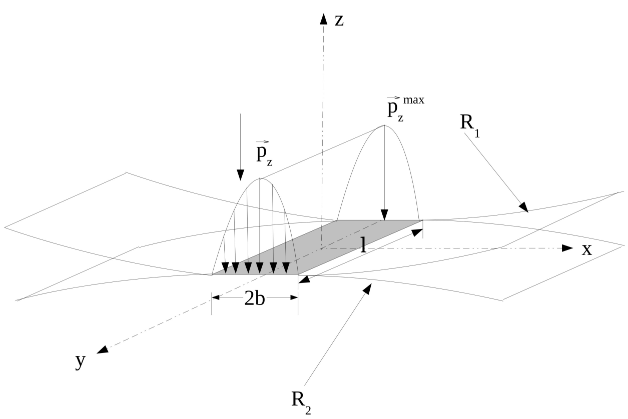


Figure 1: Schematic of the (a) twin-disc set up, (b) stress distribution over the Hertzian contact.

initially have a straight line of contact having length equal to the thickness of the disc and the width of the one-half of the contact strip is expressed by;

$$b = 2\sqrt{\frac{1-\nu^2}{\pi}\frac{P(\frac{1}{E_1}+\frac{1}{E_2})}{l(\frac{1}{R_1}+\frac{1}{R_2})}} \tag{1}$$

where,
ν = Poisson's ratio for the materials pressed against each other,
P = applied force on the cylinders,

E_1, E_2 = modulus of elasticity of the materials under compression,
R_1, R_2 = radii of the compressed cylinders, and
l = length of cylinders.

Under plain strain condition, the normal force distribution, $p_z(x)$ and the tangential force distrinution, $p_x(x)$ over the Hertzian contact zone (-b ⩽ x ⩽ b) is exprressed as ;

$$p_z(x) = \begin{cases} \frac{2p_n}{\pi b^2}\sqrt{b^2 - x^2}, & |x|<b, \\ 0, & |x| \geqslant b, \end{cases} \tag{2}$$

This gives raise to a parabolic distribution of the normal force over the Hertzian contact width which drops down to zero at both the boundaries.

$$p_x(x) = \begin{cases} 0, & |x| \geqslant b, \\ -sgn(s_1)\frac{2\mu p_n}{\pi b^2}\sqrt{b^2 - x^2}, & |x| \leqslant b, |x-b+b_1|>b_1, \\ -sgn(s_1)\frac{2\mu p_n}{\pi b^2}[\sqrt{b^2 - x^2} - \sqrt{b_1^2 - (x-b-b_1)^2}], & |x| \leqslant b, |x-b+b_1| \leqslant b_1. \end{cases} \tag{3}$$

where,

$$sgn(s_1) = \begin{cases} +1, & for s_1>0, \\ 0, & for s_1 = 0, \\ -1, & for s_1<0. \end{cases}$$

s_1 is the symbolic variable, μ is the dynamic coefficient of friction, p_n is the normal load per unit length (P/l), b_1 is the half-width of the stick zone. The stick-slip ratio, ξ, is defined as the ratio between contact half-width of the stick zone to the Hertzian contact half-width ,i.e., ξ = b$_1$/b.
Now, Integrating eq.3 within the entire contact width (a,-a) results in;

$$\frac{b_1}{b} = \left(1 - \frac{p_t}{\mu p_n}\right)^{\frac{1}{2}} \tag{4}$$

where p_t is the tangential load per unit length. Substituting eq.4 in eq.3;

$$p_x(x) = \begin{cases} 0, & |x| \geqslant b, \\ -sgn(s_1)\frac{2\mu p_n}{\pi b}\sqrt{1 - \left(\frac{x}{b}\right)^2}, & -b \leqslant x<b-2b\xi \\ -sgn(s_1)\frac{2\mu p_n}{\pi b}\left[\sqrt{1 - \left(\frac{x}{b}\right)^2} - \sqrt{\xi^2 - (\xi + \frac{x}{b} - 1)^2}\right], & b - 2b\xi \leqslant x \leqslant b. \end{cases} \tag{5}$$

normal load per unit contact half-width, p_n/b, can be taken as constant for the given pair of rollers and experimental condition. The coefficient of friction, μ during stable rolling/sliding regime are to be considered during calculations. These set of equations are numerically solved by writting a code in C language for various values of ξ and distribution of tangential force, $p_x(x)$, and normal force, $p_x(z)$ can be calculated.

3 Calculation of stresses in the x-z plane

The normal and shear stresses due to the distributed normal and tangential force acting on the Hertzian contact width is expressed as [1, 2];

$$\sigma_x(x,z) = -\frac{2z}{\pi}\int_{-b}^{b}\frac{p_z(s).(x-s)^2}{[(x-s)^2+z^2]^2}\mathrm{ds} - \frac{2}{\pi}\int_{-b}^{b}\frac{p_x(s).(x-s)^3}{[(x-s)^2+z^2]^2}\mathrm{ds} \tag{6}$$

$$\sigma_z(x,z) = -\frac{2z^3}{\pi}\int_{-b}^{b}\frac{p_z(s)}{[(x-s)^2+z^2]^2}\mathrm{ds} - \frac{2z^2}{\pi}\int_{-b}^{b}\frac{p_x(s).(x-s)}{[(x-s)^2+z^2]^2}\mathrm{ds} \tag{7}$$

$$\tau_{xz}(x,z) = -\frac{2z^2}{\pi}\int_{-b}^{b}\frac{p_z(s).(x-s)}{[(x-s)^2+z^2]^2}\text{ds} - \frac{2z}{\pi}\int_{-b}^{b}\frac{p_x(s).(x-s)^2}{[(x-s)^2+z^2]^2}\text{ds} \tag{8}$$

substituting $p_z(x)$, $p_x(x)$ and $x/b = i$, $z/b = j$ and $s/b = t$;

$$\sigma_x(i,j) = -\frac{4p_n}{b\pi^2}\Bigg[jI_{x1} + \mu I_{x2}(\xi) + \mu I_{x3}(\xi)\Bigg] \tag{9}$$

$$\sigma_z(i,j) = -\frac{4p_n j^2}{b\pi^2}\Bigg[jI_{z1} + \mu I_{z2}(\xi) + \mu I_{z3}(\xi)\Bigg] \tag{10}$$

$$\tau_{xz}(i,j) = -\frac{4p_n j}{b\pi^2}\Bigg[jI_{xz1} + \mu I_{xz2}(\xi) + \mu I_{xz3}(\xi)\Bigg]. \tag{11}$$

where,

$$I_{x1} = \int_{1}^{1}\frac{\sqrt{1-t^2}(i-t)^2}{[(i-t)^2+j^2]^2}\text{dt}, \tag{12}$$

$$I_{x2}(\xi) = \int_{-1}^{1-2\xi}\frac{\sqrt{1-t^2}(i-t)^3}{[(i-t)^2+j^2]^2}\text{dt}, \tag{13}$$

$$I_{x3}(\xi) = \int_{1-2\xi}^{1}\frac{[\sqrt{1-t^2}-\sqrt{\xi^2-(\xi+t-1)^2}](i-t)^3}{[(i-t)^2+j^2]^2}\text{dt}, \tag{14}$$

$$I_{z1} = \int_{-1}^{1}\frac{\sqrt{1-t^2}}{[(i-t)^2+j^2]^2}\text{dt}, \tag{15}$$

$$I_{z2}(\xi) = \int_{-1}^{1-2\xi}\frac{\sqrt{1-t^2}(i-t)}{[(i-t)^2+j^2]^2}\text{dt}, \tag{16}$$

$$I_{z3}(\xi) = \int_{1-2\xi}^{1}\frac{[\sqrt{1-t^2}-\sqrt{\xi^2-(\xi+t-1)^2}](i-t)}{[(i-t)^2+j^2]^2}\text{dt}, \tag{17}$$

$$I_{xz1} = \int_{-1}^{1}\frac{\sqrt{1-t^2}(i-t)}{[(i-t)^2+j^2]^2}\text{dt}, \tag{18}$$

$$I_{xz2}(\xi) = \int_{-1}^{1-2\xi}\frac{\sqrt{1-t^2}(i-t)^2}{[(i-t)^2+j^2]^2}\text{dt}, \tag{19}$$

$$I_{xz3}(\xi) = \int_{1-2\xi}^{1}\frac{[\sqrt{1-t^2}-\sqrt{\xi^2-(\xi+t-1)^2}](i-t)^2}{[(i-t)^2+j^2]^2}\text{dt}, \tag{20}$$

Having calculated σ_x, σ_z and τ_{xz}, the two principle stresses $\sigma_{1,xz}$ and $\sigma_{2,xz}$ are exprressed as;

$$\sigma_{1,xz} = \frac{\sigma_x+\sigma_z}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_z}{2}\right)^2 + \tau_{xz}^2} \tag{21}$$

$$\sigma_{2,xz} = \frac{\sigma_x+\sigma_z}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_z}{2}\right)^2 + \tau_{xz}^2} \tag{22}$$

These set of integrations are numerically solved in a program written in C programming language.

4 Input and output parameters

A list of input parameters to be called from an **input_data.txt** file are listed below in Table 1. The list of the output parameters, generated in an output file is listed in Table 2.

Table 1: List of input parameters for the code.

Parameter, unit	variable type
Poisson's ratio	double
Load, N	double
Overlap length, mm	double
Young's modulus, disc 1, GPa	double
Young's modulus, disc 2, GPa	double
Radius, disc 1, mm	double
Radius, disc 2, mm	double
Coefficient of friction	double
1-(%Slip/100)	double

Table 2: List of input parameters for the code.

Parameter, unit	variable type
Distance in x-direction, mm	double
Distance in z-direction, mm	double
Tractionals stress, σ_x, MPa	double
Tractional force/Nornal load, σ_x.b/P$_{normal}$	double
Normal stress, σ_z, MPa	double
Normal force/Nornal load, σ_z.b/P$_{normal}$	double
Shear stress, τ_{xz}, MPa	double
Shear force/Nornal load, τ_{xz}.b/P$_{normal}$	double

5 Accuracy limits

The stress values are accurate to the errors equivalent to that of the input values.

6 Key words

Hertzian contact stress, rolling contact, %slip in rolling/sliding.

7 Sample program

To be submitted in a separate ASCII text format. The code has been tested on Windows/Linux/Mac OSx platforms and found to work perfectly. An example of the output of the code, plotted using GNUplot is shown in Fig. 2.

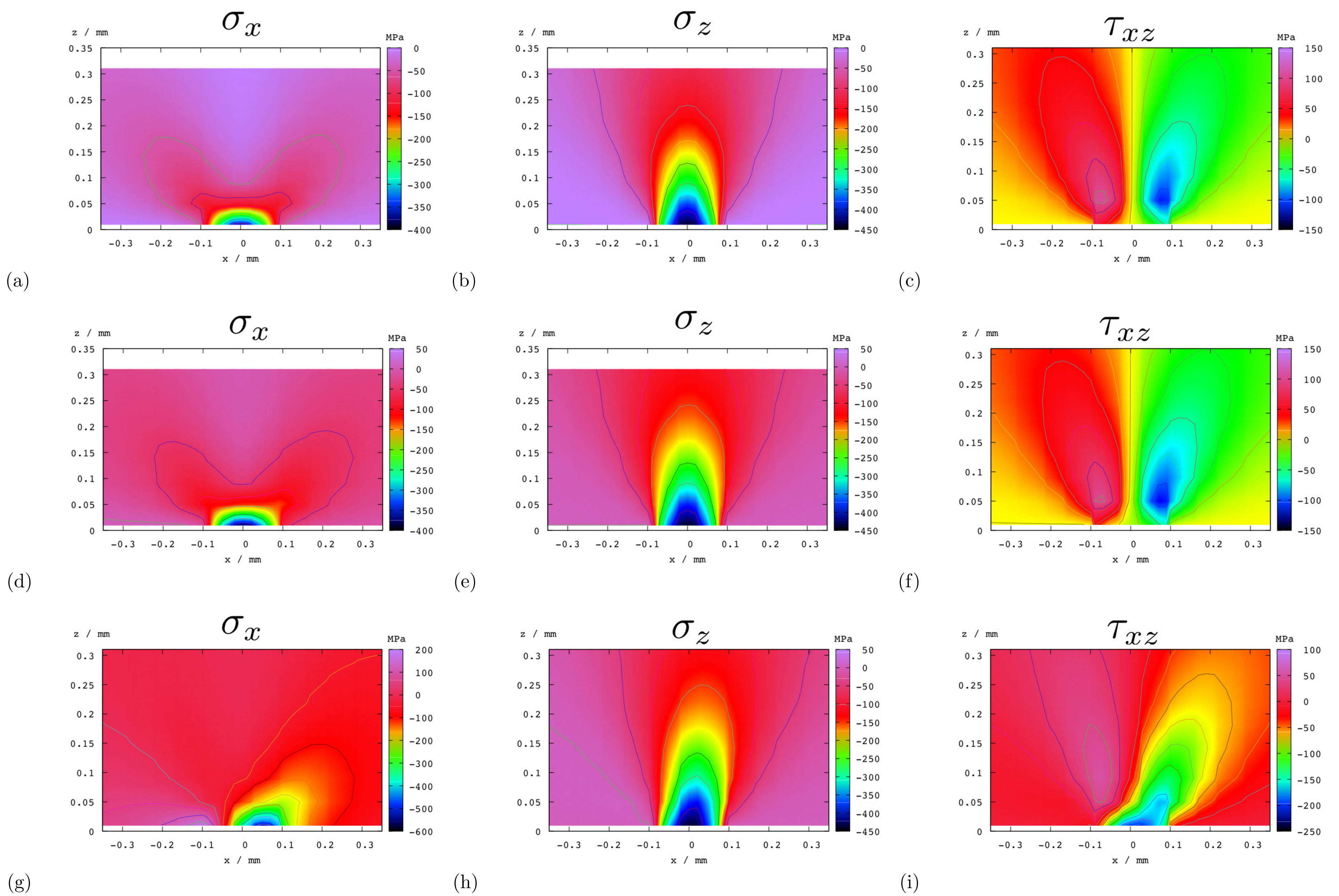


Figure 2: Calculation of tractional stress σ_x, normal stress σ_z and shear stress τ_{xz} for $l = 5\,\text{mm}$. (a-c) Assuming perfect rolling, (d-f) roll-slide parameter $\xi = 0.95$, assuming marginal slip, and (g-i) assuming perfect sliding of rolling/sliding cylinders.

8 Notification of the use of the code

The code has been used and cited in article titled *Dry rolling/sliding wear of nanostructured bainite* by S. Das Bakshi, A. Leiro, B. Prakash and H. K. D. H. Bhadeshia, submitted in Wear.

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