

absorbs carbon must equal that carried away from the  $\theta/\gamma$  interface by diffusion:

$$\frac{dz^\theta}{dt}(c^{\theta\gamma} - c^{\gamma\theta}) \approx -D_C^\gamma \frac{c^{\gamma\alpha} - c^{\gamma\theta}}{z^\alpha - z^\theta} \quad \text{for } 0 \leq z^\theta \leq S_I \times \frac{0.76 - 0.02}{6.67 - 0.02} \quad (1.3a)$$

where the numerical term on the right hand side is the fraction of cementite in pearlite of eutectoid composition,  $D_C^\gamma$  is the diffusion coefficient of carbon in austenite,  $z^\theta$  and  $z^\alpha$  represent the positions of the moving interfaces, between which the gradient on the right is defined and assumed to be constant. The distance  $z^\alpha - z^\theta$  will initially be less than  $S_I$ .

The diffusion coefficient of carbon in ferrite [15] is much greater than in austenite [16];

$$D_C^\gamma \{750^\circ\text{C}, 0.76\text{C wt}\%\} = 0.16 \times 10^{-11} \text{ m}^2 \text{ s}^{-1}$$

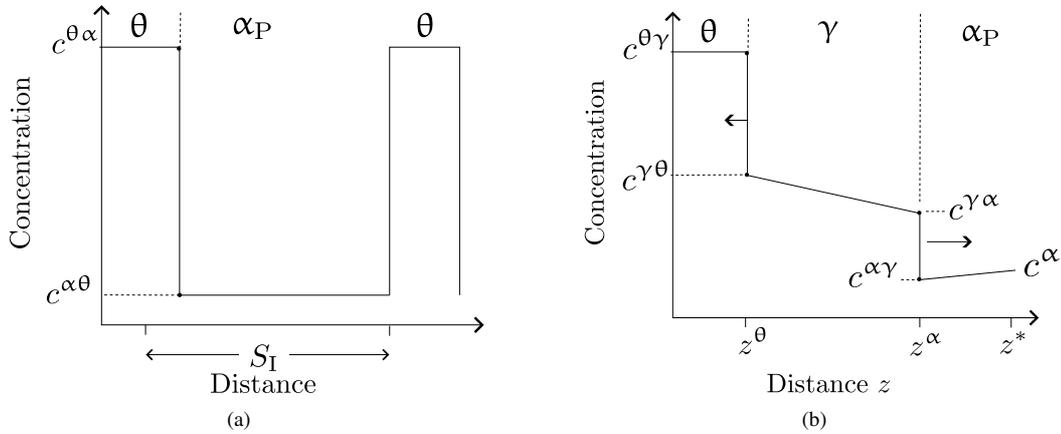
$$D_C^\alpha \{750^\circ\text{C}\} = 0.16 \times 10^{-9} \text{ m}^2 \text{ s}^{-1}$$

so assuming that the gradient of concentration in the ferrite is uniform,

$$\frac{dz^\alpha}{dt}(c^{\gamma\alpha} - c^{\alpha\gamma}) = D_C^\alpha \frac{c^\alpha - c^{\alpha\gamma}}{z^* - z^\alpha} \quad (1.3b)$$

where  $z^* \approx S_I \times (6.67 - 0.76)/(6.67 - 0.02)$  is the thickness of the ferrite lamella, calculated by applying the lever rule at  $T_E$ , and assuming that there is no austenite growing from the opposite side of the lamella. Equation ?? can be integrated to show that

$$z^* z^\alpha - \frac{(z^\alpha)^2}{2} = D_C^\alpha \frac{c^\alpha - c^{\alpha\gamma}}{c^{\gamma\alpha} - c^{\alpha\gamma}} t \quad \text{for } z^\alpha \leq z^*. \quad (1.3c)$$



**Figure 1.4** (a) The distribution of concentration prior to heating.  $S_I$  is the interlamellar spacing. (b) Schematic representation of the growth of austenite in a direction  $\pm z$  which is normal to the pearlite lamellae.  $z^\theta$  and  $z^\alpha$  represent the time-dependent locations of the  $\theta/\gamma$  and  $\alpha/\gamma$  interfaces respectively. The concentration  $c^\alpha$  is that of the ferrite at ambient temperature, now superheated to  $T > A_{e1}$ .

Equations 1.3a and 1.3b are coupled. When solved using a stepwise numerical procedure, they show that the transformation of the cementite and of ferrite is essentially simultaneous in the sense that both phases are replaced by austenite at roughly the same instant.

It is worth emphasising that the theory presented assumes diffusion-controlled growth, but given the very small starting carbon concentration in the  $\alpha_P$  prior to heating, it could be the case that the growth into ferrite is interface controlled.