Answer Sheet 1

The terminology used here is as in the lecture notes.

1. Consider equilibrium between two phases $\alpha$ and $\gamma$ in a binary alloy. Since the compositions of these phases are not identical, $x_{\alpha\gamma} \neq x_{\gamma\alpha}$, it follows that concentration gradients exist and yet there will be no diffusion since the phases are at equilibrium.

2. Given that $G = \mu_A(1 - x) + \mu_Bx$, it follows that $\frac{\partial G}{\partial x} = \mu_B - \mu_A$.

3. Coiling and uncoiling of molecular chains, vibration of side groups, rotation about covalent bonds.

4. In ordered crystals, $A$ atoms prefer to be next to $B$ atoms: the enthalpy of ordering $\Delta H_O$ is negative. However, there is a decrease in entropy on ordering so $-T\Delta S_O$ is positive. The latter term dominates at high temperatures, making $\Delta G_O$ positive and hence favouring disorder (random distribution of atoms).

5. An ideal solution is one in which the atoms are randomly mixed at all temperatures. The probability of finding an $A$ atom next to a $B$ atom (or vice versa) in an equiatomic ideal solution is

$$p_{AB} = 2x(1 - x) = 0$$

since $x$ is the probability of finding a $B$ atom and $x(1 - x)$ is that of finding an $A$ atom next to a $B$ atom.

6. The task is to calculate the equilibrium carbon concentration at any point given a fixed manganese concentration gradient in austenite. The activity ($a$) of carbon will tend to become uniform:

$$\ln\left\{a_C^0\right\} = \ln\left\{a_C^{Mn}\right\}$$

$$\ln\left\{\Gamma_C^0\right\} + \ln\left\{x_C^0\right\} = \ln\left\{\Gamma_C^{Mn}\right\} + \ln\left\{x_C^{Mn}\right\}$$

where $a_C^0$ is the activity of carbon at zero Mn, $a_C^{Mn}$ is the activity of carbon at a finite Mn concentration, $x_C^0$ and $x_C^{Mn}$ are the corresponding mole fractions of carbon, $\Gamma_C^0$ and $\Gamma_C^{Mn}$ are the corresponding activity coefficients. The activity coefficients can be expanded as follows (Kirkaldy and Baganis, Metall. Trans. 9A, 1978, 495):

$$\ln\left\{\Gamma_C\right\} = 8.1 \times x_C - 5 \times x_{Mn}$$

where $x_{Mn}$ is the concentration of manganese. It follows that

$$(8.1 \times x_C^0) + \ln\left\{x_C^0\right\} = (8.1 \times x_C^{Mn} - 5 \times x_{Mn}) + \ln\left\{x_C^{Mn}\right\}$$

$$\ln\left\{x_C^0\right\} - \ln\left\{x_C^{Mn}\right\} = (8.1 \times [x_C^{Mn} - x_C^0] - 5 \times x_{Mn})$$

Writing $[x_C^0 - x_C^{Mn}] = \Delta x$, we get

$$\ln\left\{1 + \frac{\Delta x}{x_C^{Mn}}\right\} = -8.1 \times \Delta x - 5 \times x_{Mn}$$

which for small $\Delta x$ becomes

$$x_C^0 - x_C^{Mn} \equiv \Delta x = \frac{-5x_{Mn}}{8.1 + \frac{1}{x_C^{Mn}}}$$

Suppose we have 1 wt% C and the manganese concentration ranges from 0–5 wt%. 1 wt% C is about 0.05 mole fraction of carbon. Setting $x_C^{Mn} \simeq 0.05$, $x_{Mn} \simeq 0.05$ (since we have 5 wt% Mn), we see that $\Delta x = 0.0089$ or 0.18 wt%. The carbon concentration in the Mn–rich region will therefore be higher by about 0.18 wt%.