Answers: Bayes' Rule

1.

$$\begin{aligned}\mathrm{P}(GG) &= \mathrm{P}(GG|M)\mathrm{P}(M) + \mathrm{P}(GG|D)\mathrm{P}(D)\\ &= \frac{1}{2}\mathrm{P}(M) + \frac{1}{4}[1 - \mathrm{P}(M)]\end{aligned}$$

Therefore,

$$\mathrm{P}(M) = 4\mathrm{P}(GG) - 1 = 0.6$$

2.

$$\begin{aligned}\mathrm{P}(L|Y) &= \frac{\mathrm{P}(Y|L)\mathrm{P}(L)}{\mathrm{P}(Y|L)\mathrm{P}(L) + \mathrm{P}(Y|C)\mathrm{P}(C)}\\ &= \frac{0.55 \times 0.52}{0.55 \times 0.52 + 0.85 \times 0.48}\\ &= 0.41\end{aligned}$$

$$\begin{aligned}\mathrm{P}(C|Y) &= \frac{\mathrm{P}(Y|C)\mathrm{P}(C)}{\mathrm{P}(Y|L)\mathrm{P}(L) + \mathrm{P}(Y|C)\mathrm{P}(C)}\\ &= \frac{0.85 \times 0.48}{0.55 \times 0.52 + 0.85 \times 0.48}\\ &= 0.59\end{aligned}$$

3.

$$\mathrm{P}(L|P) = 26/232 = 0.11 \qquad \mathrm{P}(R|P) = 206/232 = 0.89$$

$$\mathrm{P}(P|L) = 26/48 = 0.54 \qquad \mathrm{P}(P|R) = 206/259 = 0.80$$

The test $\mathrm{P}(L|P) < \mathrm{P}(R|P)$ is irrelevant. What we need is to see is what the probability of passing given that the student is left–handed is, and to compare against the corresponding probability of passing given that the student is right–handed, *i.e.* $\mathrm{P}(P|L) < \mathrm{P}(P|R)$. Since the latter is true, there appears to be discrimination.

4. Denote an invalid result by I and a valid result by V.

$$\begin{aligned}\mathrm{P}(A|I) &= \frac{\mathrm{P}(I|A)\mathrm{P}(A)}{\mathrm{P}(I|A)\mathrm{A}(L) + \mathrm{P}(I|B)\mathrm{P}(B)}\\ &= \frac{0.10 \times 0.40}{0.10 \times 0.40 + 0.05 \times 0.60} = \frac{4}{7}\end{aligned}$$

Notes

Notice that it is sometimes assumed in the answers that the fraction of observations is also the probability. Suppose we throw an unbiased coin four times, and obtain a head only once, then with this assumption the probability of obtaining a head would be a quarter. This clearly is incorrect.